Uncertainty in Procurement Contracting with Time Incentives*

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Abstract

This paper studies cost-plus-time (A+B) procurement contracting with time incentives in the highway construction industry. In the presence of construction uncertainty, the contractor's actual completion time may deviate from the bid completion time, and the A+B contract design is not ex-post efficient. Using data from highway procurement contracts in California, we show that an ex-post efficient lane rental contract would reduce the social cost by $41.39 million (43.11 percent) on average. Moreover, the average commuter cost would decrease by $62.06 million (78.96 percent), suggesting a substantial reduction in the construction externality to commuters from lane rental contracts.

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1. Introduction

In many procurement contracts, the buyer cares about multiple attributes and uses scoring auctions to select the seller. Moreover, post-auction uncertainty is an inherent component in many auctions such as auctions for oil tracts, timber, and construction (McAfee and McMillan, 1986; Esö and White, 2004; Lewis and Bajari, 2014; Bajari et al., 2014; Luo et al., 2018a; An and Tang, 2019; Luo and Takahashi, 2019; Bhattacharya et al., 2020). One prominent example is that in highway construction procurement, buyers often care about both construction cost and commuter cost, where the commuter cost is represented by the negative externality to commuters during construction. In these situations, buyers use scoring auctions to choose the contractor. The bid score in scoring auctions is a weighted sum of bids of construction cost and completion time, and the bidder with the lowest score wins the contract. During the construction stage, however, the contractor usually faces unexpected shocks, including technical and logistical shocks (Perry and Hayes, 1985; Lewis and Bajari, 2014; Bajari et al., 2014; An and Tang, 2019; Luo and Takahashi, 2019). These unexpected shocks may lead to late completion of the project. Since accelerating construction is costly for contractors, the procurer provides time incentives to reward early completion and punish late completion to induce the contractor to shorten the actual completion time. Therefore, designing efficient multi-attribute mechanisms in the presence of uncertainty has attracted considerable attention from theorists

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and practitioners in recent decades.

This paper investigates the efficiency of multi-attribute mechanisms in the presence of uncertainty by using highway procurement contracts from the California Department of Transportation (Caltrans). The contract awarded through scoring auctions is a cost-plus-time contract, so-called A+B contract, in which A refers to the bid cost and B refers to the bid completion time. Motivated by the empirical relevance of uncertainty in highway construction, we theoretically analyze how the presence of construction uncertainty affects the efficiency of A+B contracts with time incentives. Then, we identify and estimate the model primitives. Using data on highway procurement contracts, we conduct counterfactuals to quantify the differences in efficiency between A+B and alternative contracts.

Our model builds on the literature on scoring auctions with time incentives by incorporating ex-post uncertainty. The ex-post uncertainty can cause number of actual working days to deviate from the number bid. In our data, more than two-thirds of contracts are completed earlier or later than the bid days. The deviation of actual days from the bid days is because in the presence of ex-post uncertainty, the actual working days is a trade-off between the cost reduction and the penalty for late completion or between the cost increase and the reward for early completion. This deviation results in the ex-post inefficiency of A+B contracts. If there was no ex-post uncertainty, however, the project would always be completed on time. Therefore, the time incentives in A+B contracts alone cannot lead to the deviation of actual days from the bid days, and in the absence of ex-post uncertainty the A+B contract can be ex-post efficient (Lewis and Bajari, 2011).

We show that the model primitives, including the contractor's cost function, the distribution of the contractor's private cost type, and the uncertainty distribution, can be identified from the observed bids for cost and completion time, the actual working days, and the equilibrium implications of the model. Our identification strategy is outlined in several steps: first, by following Guerre et al. (2000), we explore the one-to-one mapping between the bid score and the bidder's pseudo-type to back out the pseudo-type, which is simply an increasing transformation of the bidder's cost type. This one-to-one mapping is analogous to that between the bid cost and cost type in the standard first-price auctions wherein the bid cost is increasing in cost type.

Next, we exploit the quantile relationship between the observed bid days and the recovered pseudo-type to identify the parameters of the cost function, where the quantile relationship is implied by the fact that the pseudo-type and the bid days are both strictly increasing in cost type. In the next step, we recover the cost types using the observed bid days and, in turn, identify the cost types' distribution. Similarly, we use the observed actual days to back out the uncertainties associated with contracts that are completed early or late, and then identify the parameters of the uncertainty distribution. Following the identification strategy, we propose a multistep semiparametric estimation procedure.

Motivated by the pattern in the deviation of actual days from bid days in the data, we apply our model to evaluate the social welfare of highway A+B procurement contracts in California. Using the model estimates, we compare the welfare performance between A+B and lane rental contracts. A lane rental contract, in which the contractor pays a fixed fee for each day the lanes are occupied, is designed to reduce completion time and commuter costs in heavily populated areas or on busy roads (Srinivasan and Harris, 1991; Herbsman and Glagola, 1998). Although both lane rental and A+B contracts are designed primarily to reduce construction time, there is no consensus among researchers and practitioners regarding which contract mechanism is preferred (Strong, 2006).

To alleviate the discrepancy in contract design preferences, we study the differences in efficiency between A+B and lane rental contracts. First, we prove that the lane rental contract is ex-post efficient in the presence of ex-post uncertainty when the disincentive for late completion is equal to the daily commuter cost. In the counterfactual analysis, we find that the average social cost under ex-post efficient lane rental contracts would be $41.39

million (43.11 percent) lower than that under A+B contracts, where the social cost is the sum of the construction cost and the commuter cost. In particular, the average commuter cost under lane rental contracts would decrease by $62.06 million (78.96 percent). This suggests a substantial reduction in the construction externality to commuters. However, this substantial commuter gain entails higher construction costs due to the smaller number of working days in lane rental contracts.

This paper has major qualitative differences from two related papers by Lewis and Bajari (2011, 2014). Lewis and Bajari (2011) offer the first attempt to analyze A+B contracting with time incentives. In the absence of post-auction uncertainty, the project is finished on time in equilibrium, and this contracting can be ex-post efficient. Our model builds on their model by incorporating ex-post uncertainty to explain the empirical pattern in the deviation of actual days from bid days. This deviation implies that A+B contracting with time incentives is no longer ex-post efficient in the presence of ex-post uncertainty.

Additionally, Lewis and Bajari (2014) focus on how ex-post construction uncertainty affects the effort exerted by the contractor in the construction stage. However, they do not consider adverse selection in the bidding stage. In contrast, our model is able to analyze both the ex-ante and the ex-post efficiency of the contracting by integrating the prospective contractors' bidding behavior with the winning contractor's ex-post strategic choice of working days.

Moreover, our work is motivated by different policy issues from Lewis and Bajari (2011, 2014). These two papers investigate the welfare gains from switching the standard bid-cost-only auctions to A+B contracts with time incentives and to lane rental contracts, respectively. They provide empirical evidence for the superiority of both A+B and lane contracts over standard auctions. In contrast, we compare the welfare difference between A+B and lane rental contracts. The counterfactuals indicate that lane rental contracts can lead to smaller commuter costs and social costs than A+B contracts, while A+B contracts are preferred in terms of reducing construction cost.

We contribute to the literature in several other aspects. First, this paper contributes to a growing body of empirical literature on scoring auctions in the presence of post-auction uncertainty. Aside from A+B scoring auctions, another widely used scoring auction is the unit-price auction in which bidders submit unit-price bids for each item/input required to complete a project. The winning bidder is determined by the lowest score, where the score is the sum of unit bids multiplied by item quantity estimates provided by the procurer. The winner is then paid based on the quantities actually used upon completing the project. The ex-post adjustments on inputs during project implementation can affect the bidding behavior or the procurement cost.

When the adjustment costs of contracted items are covered by the procurer, contractors strategically choose their unit-price bids by bidding high on underestimated items and bidding low on overestimated items. This skewed bidding is driven by the consideration that positive adjustments increase bidder revenue, while negative adjustments reduce bidder revenue. Empirical evidence shows that the skewed bidding can result in higher procurement costs through the selection of inefficient contractors (Luo and Takahashi, 2019). On the other hand, when the contractor and the procurer renegotiate compensation in the adjustment costs of uncontracted items, the renegotiation imposes significant adaptation costs, and contractors have substantial bargaining power in renegotiation (Bajari et al., 2014; An and Tang, 2019). In comparison, ex-post uncertainty in A+B contracting with time incentives can result in the ex-post inefficiency of contracting due to the contractor's strategic choice of the actual working days during project implementation.

Second, our work is related to the literature on single-attribute auctions in the presence of ex-post uncertainty. These studies analyze how the presence of ex-post uncertainty affects bidding strategy in single-attribute auctions (Hendricks and Porter, 1988; Haile, 2001; Esö and White, 2004). In comparison, this paper examines how ex-post uncertainty affects the equilibrium and efficiency of multi-attribute auctions.

Third, this paper contributes to a broader stream of literature on multi-attribute auctions by introducing post-auction uncertainty. The existing literature on multi-attribute auctions without post-auction uncertainty includes theoretical work (Che, 1993; Branco, 1997; Fang and Morris, 2006; Asker and Cantillon, 2008; Wang and Liu, 2014) and empirical studies (Levin and Athey, 2001; Lewis and Bajari, 2011; Koning and Van De Meerendonk, 2014; Takahashi, 2018; Huang, 2019; Krasnokutskaya et al., 2020). Our model builds on Che (1993) and Asker and Cantillon (2008) by incorporating ex-post uncertainty. Moreover, we empirically investigate the impacts of post-auction uncertainty on the bidding behavior and the efficiency of contracting.

Fourth, our work is related to a vast literature on the identification of auctions and contracts (Guerre et al., 2000; Jofre-Bonet and Pesendorfer, 2003; Krasnokutskaya, 2011; Hu et al., 2013; Gentry and Li, 2014; Li et al., 2015; Luo et al., 2018a,b; An and Tang, 2019; Zhang, 2021). The identification arguments in these papers rely crucially on the mappings between the unobserved types of agents and the contract characteristics in the data. In comparison, in addition to these one-to-one mappings, we exploit the correlation between bid days and actual days to construct a quantile relationship between pseudo-type and bid days to identify the model.

In Section 2, we analyze the equilibrium and efficiency of our model. Section 3 provides the main identification results. Section 4 introduces the background of highway A+B procurement contracts and describes the data. Section 5 defines our estimation method and reports the estimation results. In Section 6, we compare the welfare under lane rental contracts with that in the data. Section 7 concludes the paper. Proofs and other details are collected in the Appendix.

2. The Model

2.1. Setup

A risk-neutral buyer (or procurer) seeks to procure a highway project from among $N \geq 2$ potential risk-neutral bidders (or contractors). In the scoring auction, each bidder submits a bid, which is a combination of cost $p^B \in \mathcal{P} \subset \mathbb{R}_+$ and working days $x^B \in \mathcal{X} \subset \mathbb{R}_+$. Prior to the auction stage, the procurer announces three messages: (i) an engineer's estimate of the pair (p^E, x^E) for the project, where p^E and x^E are the estimates of project cost and working days, respectively; (ii) a scoring rule $s : \mathcal{P} \times \mathcal{X} \mapsto s(\mathcal{P}, \mathcal{X})$ that associates a score with a bid pair (p^B, x^B) and represents a continuous preference of the procurer over (p^B, x^B); and (iii) an incentive/disincentive scheme (r, d), where $r \in \mathbb{R}_+$ and $d \in \mathbb{R}_+$ are the daily cash bonus for early completion and the daily cash penalty for late completion, respectively. We maintain $r < d$ to follow the practice used by Caltrans and because it is consistent with our data (e.g., Lewis and Bajari, 2011).

There is an ex-ante private cost type θ_i for bidder $i \in \mathcal{N} = \{1, \cdots, N\}$, which is drawn independently from a distribution $F_\Theta(\cdot)$ with density $f_\Theta(\cdot)$ and support $\mathcal{S}_\Theta \subset \mathbb{R}_+$. Type θ_i reflects contractor i's innate cost, such as managerial capacity, expertise in working on a tight schedule, or relationships with input suppliers or subcontractors. In the broad literature on hidden information (e.g., Gagnepain and Ivaldi, 2002; Rogerson, 2003; Perrigne and Vuong, 2011; Gagnepain et al., 2013), the cost type can capture firms' cost-related productivity, including some flexibility related to their management skills, though other types of flexibility may be unrelated to productivity.

The uncertainty ε_i ex-post auction for bidder $i \in \mathcal{N}$ is distributed as $F(\cdot)$ with density $f(\cdot)$ and support $\mathcal{S}_\varepsilon \subset \mathbb{R}_+$. Uncertainty captures the unexpected shocks (such as productivity shocks and input delays) encountered by the contractor during the construction stage. Empirical evidence for the relevance of construction uncertainty in highway procurement contracts can be found in Bajari et al. (2014) and Lewis and Bajari (2014), among others. Following the related literature (e.g., Esö and White, 2004; Luo et al., 2018a), the ex-post uncertainty ε_i is assumed to be independent of all θ_j, including θ_i.[2] However, we allow ε_i to be correlated among bidders or even identical, in the case of macroeconomic or other exogenous shocks common to all bidders. Upon drawing the private cost type θ_i, bidder i quotes a sealed bid pair (p_i^B, x_i^B). The contract is awarded to the bidder with the lowest score, where the score is calculated according to the announced scoring rule.

As in Esö and White (2004), the *deterministic* construction cost for bidder i is $c(x_i, \theta_i)$, which represents the cost of completing the project in days x_i for a contractor of type θ_i, and the *actual* construction cost is the deterministic cost times the construction uncertainty, i.e., $\varepsilon_i \cdot c(x_i, \theta_i)$. This multiplicative structure is similar to that in Bajari et al. (2014), who evaluate the adaptation cost caused by ex-post construction shocks in California's highway procurement contracts. Note that if ε_i is constant with $\varepsilon_i = 1$ for any $i \in \mathcal{N}$, our cost specification reduces to the case without uncertainty as in Lewis and Bajari (2011). As will be shown below, in this multiplicative specification, the actual working days can deviate from the bid days in equilibrium.

An alternative specification for the *actual* construction cost could be the *deterministic* construction cost plus the ex-post uncertainty. However, in equilibrium, the actual working days would always equal the bid days, as the marginal *actual* construction cost of working days does not depend on uncertainty. This is not consistent with the pattern in our data, where actual working days can deviate from bid days. This echoes Luo et al. (2018a), who note that, under the above additive structure, the equilibrium bid in the

[2]This independence of uncertainty from cost type can be interpreted as conditional on the observed characteristics of the project, contractor, or economic environment (e.g., Lewis and Bajari, 2014). In the empirics, we allow for correlation between uncertainty and cost type through observed heterogeneous characteristics across bidders.

first-price sealed-bid auction with ex-post uncertainty is the same as that without ex-post uncertainty. Therefore, we use the multiplicative structure to rationalize the deviation pattern in our data.[3]

After winning the contract, the winning bidder's *total* cost of completing the project is the sum of the *actual* construction cost $\varepsilon \cdot c(x^A, \theta)$ and the incentive cost $K(x^A, x^B, r, d)$, where x^A represents the actual working days. We suppress the dependence of variables on the winner's identity for simplicity. The actual working days x^A may differ from the bid days x^B. This is because under time incentives, the contractor may adjust his initial implementation plan in response to the realized uncertainty during the construction stage. The incentive cost $K(x^A, x^B, r, d)$ represents the incentive scheme determined by the procurer. As used by Caltrans, the contractor faces a punishment $d \cdot (x^A - x^B)$ if the actual working days x^A exceed the bid days x^B or receives a reward $r \cdot (x^B - x^A)$ if the project is completed in less than the bid days. The model primitives $\mathcal{M} = [F_\Theta(\cdot), F(\cdot), c(\cdot, \cdot)]$ are common knowledge for all players but unknown to the econometrician. Finally, our model can be viewed as the second stage of a game with entry in the first stage in the spirit of Levin and Smith (1994) and Luo et al. (2018a), where N is the number of entrants.

2.2. Equilibrium

We maintain the standard assumptions to analyze the equilibrium of A+B contracting with time incentives in the presence of uncertainty. We focus on the symmetric pure strategy Bayesian Nash Equilibrium (psBNE), where a psBNE consists of a bidding strategy $(p^{B^*}(\theta), x^{B^*}(\theta))$ and an actual working days strategy $x^{A^*}(\theta, \varepsilon)$ for any (θ, ε). For a generic function $g(\cdot)$ with more than one argument, we use $g_i(\cdot)$ to denote the first-order deriva-

[3] Under the additive structure of the *actual* construction cost, if bidders are risk-averse, the actual working days might deviate from the bid days. However, this is beyond the scope of this paper and left to future research.

tive with respect to its i-th argument and $g_{ij}(\cdot)$ to denote the second-order derivative with respect to its i-th and j-th arguments.

Assumption 1. *The deterministic cost function satisfies* $c(\cdot,\cdot) \geq 0$*,* $c_1(\cdot,\cdot) < 0$*,* $c_2(\cdot,\cdot) > 0$*,* $c_{11}(\cdot,\cdot) > 0$*, and* $c_{12}(\cdot,\cdot) < 0$*.*

Assumption 1 imposes standard conditions on the cost function $c(\cdot,\cdot)$ (e.g., Laffont and Tirole, 1993; Lewis and Bajari, 2011). In particular, Lewis and Bajari (2011) assume that the cost function is decreasing in working days when the number of working days is smaller than the engineer's estimate, which corresponds to the efficient scale of construction. Since the procurer requires that the number of bid days be less than or equal to the engineer's estimate, it seems plausible to assume that the construction cost decreases in working days. The decreasing monotonicity of the cost function implies that construction acceleration is costly. As a result, it motivates the procurer to reward contractors for early completion and punish them for late completion. This decreasing monotonicity is consistent with Lewis and Bajari (2011), who assume that the acceleration cost, defined as a function of the deviation in the engineer's days from the working days, is increasing in deviation. Moreover, the condition that the marginal cost of working days is decreasing in type implies that a less efficient (larger θ) contractor enjoys a larger cost reduction induced by one additional working day. Therefore, it may provide higher-powered incentives for less efficient contractors to bid more working days. As shown later, the equilibrium number of bid days is increasing in type.

Next, we specify a linear-in-price scoring rule, which is widely used in highway procurement contracts by the Department of Transportation (DoT) in the United States.[4]

[4]These states include California, Delaware, Idaho, Massachusetts, Oregon, Texas, Utah, and Virginia. In addition, other scoring rules are used, such as the price-over-quality ratio rule in Alaska and Colorado. We restrict our analysis to the linear-in-price rule since it is consistent with the data in hand.

The scoring rule is given by

$$s(p^B, x^B) = p^B + c_u \cdot x^B, \tag{1}$$

where the user cost $c_u \geq 0$ measures the time value of the externality entailed by construction. The quasilinear scoring rule (1) implies that we can use the one-dimensional private cost type to rationalize the nonmonetary attribute: the bid days.[5] Given that the bidder's payoff is additively separable in bid cost and using arguments similar to those in Che (1993) and Asker and Cantillon (2008), the equilibrium bid days can be first obtained by solving a maximization problem in which only the private type of the bidder matters. The equilibrium bid cost is determined by maximizing the bidder's payoff given his private type and the equilibrium bid days.[6]

We use backward induction to analyze the model equilibrium. In the second stage of construction, we analyze the winning contractor's optimal actual working days given his type θ, bid pair (p^B, x^B), and uncertainty ε. Because the payoff in the construction stage is the bid cost p^B minus the total cost, the contractor chooses the optimal actual working days by minimizing his total cost,

$$\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = \underset{x^A}{\operatorname{argmin}} \{\varepsilon \cdot c(x^A, \theta) + K(x^A, x^B, r, d)\}. \tag{2}$$

Intuitively, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon)$ may not equal the bid days x^B due to the presence of ex-post uncertainty. If the uncertainty is small (large) relative to incentive r (disincentive

[5] A scoring rule is quasilinear if the monetary attribute, which is the bid cost in the model, is additively separable (Che, 1993).

[6] If the scoring rule is not quasilinear, the monetary attribute can also be relevant for the dimension of the private type. For example, in design-build scoring auctions where the score is the ratio of the price to the one-dimensional quality (that is, the scoring rule is not quasilinear), Takahashi (2018) assumes that each bidder has a two-dimensional cost type.

d), the contractor tends to complete the project earlier (later) than the bid days; if the uncertainty is mild relative to the incentive/disincentive, the contractor tends to complete the project on time. Therefore, the optimal actual working days is a trade-off between the cost reduction and the penalty for late completion or between the cost increase and the reward for early completion. The lemma below formalizes these results.

Lemma 1. *Under Assumption 1, the second-stage optimal actual working days given* $(x^B, \theta, \varepsilon)$ *satisfies*

$$\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = \begin{cases} x^r(\theta, \varepsilon) & \text{if } \varepsilon \leq \varepsilon^r(\theta, x^B), \\ x^B & \text{if } \varepsilon \in [\varepsilon^r(\theta, x^B), \varepsilon^d(\theta, x^B)], \\ x^d(\theta, \varepsilon) & \text{if } \varepsilon \geq \varepsilon^d(\theta, x^B), \end{cases} \tag{3}$$

where the two cutoff levels of uncertainty are given by

$$\varepsilon^r(\theta, x^B) = \frac{-r}{c_1(x^B, \theta)} \text{ and } \varepsilon^d(\theta, x^B) = \frac{-d}{c_1(x^B, \theta)}, \tag{4}$$

the early working days $x^r(\theta, \varepsilon)$ *and late working days* $x^d(\theta, \varepsilon)$ *are given by*

$$-\varepsilon \cdot c_1(x^r(\theta, \varepsilon), \theta) = r \text{ and } -\varepsilon \cdot c_1(x^d(\theta, \varepsilon), \theta) = d. \tag{5}$$

Moreover, $\varepsilon^r(\theta, x^B) < \varepsilon^d(\theta, x^B)$ *and* $x^r(\theta, \varepsilon) \leq x^B \leq x^d(\theta, \varepsilon)$.

Proof: See the Appendix.

Next, we use Lemma 1 to analyze the equilibrium bids of cost and completion time. Since we focus on the symmetric equilibrium, we drop the index of a bidder i for expositional simplicity. In the bidding stage, a bidder with type θ quotes a pair of cost and working days $(p^{B^*}(\theta), x^{B^*}(\theta))$ to maximize his expected payoff

$$\begin{aligned}\big(p^{B^*}(\theta), x^{B^*}(\theta)\big) &= \operatorname*{argmax}_{p^B, x^B} \Bigg\{ \bigg(p^B - \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), x^B, r, d) \Big] \bigg) \\ &\times \ \Pr\Big(\text{win} \ \Big| \ s = p^B + c_u x^B \Big) \Bigg\}, \qquad (6)\end{aligned}$$

where $\Pr(\text{win}|s = p^B + c_u x^B)$ is the conditional probability of winning the auction given bid score $s = p^B + c_u x^B$, and $\mathbb{E}_\varepsilon$ denotes the expectation with respect to ε. Following the literature on scoring auctions (e.g., Che, 1993; Asker and Cantillon, 2008), the contractor's optimization problem in (6) is equivalent to choosing the optimal score $s(v(\theta))$:

$$s(v(\theta)) = \operatorname*{argmax}_{b} \Big\{ \big(b - v(\theta)\big) \times \Pr(\text{win} \mid b) \Big\}, \qquad (7)$$

where the contractor's pseudo-type $v(\theta)$ is given by

$$v(\theta) = \min_{x^B} \Big\{ c_u x^B + \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), x^B, r, d) \Big] \Big\}. \qquad (8)$$

The equilibrium bid days $x^{B^*}(\theta)$ corresponds to the minimizer of the objective function in (8).[7] Then, the equilibrium payment is $p^{B^*}(\theta) = s(v(\theta)) - c_u x^{B^*}(\theta)$, and the equilibrium actual working days is $x^{A^*}(\theta, \varepsilon) = \widetilde{x}^{A^*}(x^{B^*}(\theta), \theta, \varepsilon)$.

In addition, the winner is the contractor with the lowest type because

$$\frac{d}{d\theta} s(v(\theta)) = s'(v(\theta)) v'(\theta) > 0, \qquad (9)$$

where $v'(\theta) > 0$ and $s'(v(\theta)) > 0$, as shown in the proof of Proposition 1, are typical properties in the auction literature (e.g., Asker and Cantillon, 2008; Krishna, 2009).

[7] The equilibrium bid days $x^{B^*}(\theta)$, if existing, must be unique because it can be shown that the objective function (8) is globally convex in x^B by using the proofs in Proposition 1.

Proposition 1. *Under Assumption 1, there exists a unique symmetric psBNE* $\big(p^{B^*}(\theta), x^{B^*}(\theta), x^{A^*}(\theta,\varepsilon)\big)$ *for the A+B contract such that for any* θ*, we have the following:*

(i) The equilibrium bid for working days is

$$x^{B^*}(\theta) = \arg\min_{x^B} \Big\{ c_u x^B + \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B,\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(x^B,\theta,\varepsilon), x^B, r, d) \Big] \Big\}. \tag{10}$$

Moreover, $dx^{B^*}(\theta)/d\theta > 0$.

(ii) The equilibrium actual number of working days is

$$x^{A^*}(\theta,\varepsilon) = \widetilde{x}^{A^*}(x^{B^*}(\theta),\theta,\varepsilon) = \begin{cases} x^r(\theta,\varepsilon) & \text{if } \ \varepsilon \le e^r, \\ x^{B^*}(\theta) & \text{if } \ \varepsilon \in [e^r, e^d], \\ x^d(\theta,\varepsilon) & \text{if } \ \varepsilon \ge e^d, \end{cases} \tag{11}$$

where in equilibrium the two cutoff levels of uncertainty are constant for any θ

$$\varepsilon^r(\theta, x^{B^*}(\theta)) = e^r \ \ and \ \ \varepsilon^d(\theta, x^{B^*}(\theta)) = e^d \tag{12}$$

with $e^r < e^d$*. Moreover,* $\partial x^r(\theta,\varepsilon)/\partial\theta > 0$*,* $\partial x^r(\theta,\varepsilon)/\partial\varepsilon > 0$*,* $\partial x^d(\theta,\varepsilon)/\partial\theta > 0$*, and* $\partial x^d(\theta,\varepsilon)/\partial\varepsilon > 0$*.*

(iii) The equilibrium bid for cost is

$$\begin{aligned} p^{B^*}(\theta) &= \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(x^{A^*}(\theta,\varepsilon),\theta) + K(x^{A^*}(\theta,\varepsilon), x^{B^*}(\theta), r, d)\Big] \\ &+ \int_\theta^{\overline{\theta}} \mathbb{E}_\varepsilon[\varepsilon \cdot c_2(x^{A^*}(t,\varepsilon),t)] \left[\frac{1-F_\Theta(t)}{1-F_\Theta(\theta)}\right]^{N-1} dt. \end{aligned} \tag{13}$$

Proof: See the Appendix.

In Part (i), the bid days $x^{B^*}(\theta)$ is increasing in type θ, which implies that for a less efficient (larger type) contractor, more working days are required to complete the project.

In Part (*ii*), the actual working days may deviate from the bid days, depending on the level of uncertainty. However, as shown in Lewis and Bajari (2011), if there were no uncertainty, the actual working days would always equal the bid days. In addition, the two cutoff levels of uncertainty in equilibrium are constant for any type. This is because the cutoff level of uncertainty depends on type through the marginal cost of bid days, while the marginal cost of bid days in equilibrium is constant for any type. This constancy of the marginal cost can be explained by the direct effect of type on the marginal cost and the indirect effect through the equilibrium bid days $x^{B^*}(\theta)$. As shown in the proof of Proposition 1, these two opposite effects cancel out when type changes, leading to the constancy of the marginal cost and hence the cutoff levels of uncertainty.

2.3. Efficiency

Similar to Lewis and Bajari (2011), the social welfare in the presence of uncertainty is given by

$$W(\theta, \varepsilon, x^A) = V_c - \varepsilon \cdot c(x^A, \theta) - c_s x^A = V_c - S_c, \tag{14}$$

where V_c is the social value of the project, $\varepsilon \cdot c(x^A, \theta)$ is the construction cost, $c_s x^A$ is the commuter cost induced by the construction externality to commuters with daily commuter cost $c_s \geq 0$, and the social cost S_c is the sum of the construction cost and commuter cost.

We say that a contract design is ex-post efficient if the completion time x^A is welfare-maximizing for all types θ and all uncertainties ε. In other words, ex-post efficiency implies that regardless of which contractor wins the contract, the winner always maximizes social welfare. A contract design is ex-ante efficient if the contract is always awarded to the bidder who generates the highest social welfare in equilibrium for all uncertainties ε. These two notions distinguish regulating the winning contractor (ex-post efficiency) from choosing that contractor (ex-ante efficiency).

First, as suggested by the deviation of actual working days from the bid days in Proposition 1, the A+B contract is not ex-post efficient in the presence of uncertainty. Intuitively, these two distinct time incentives for early and late completion cannot both be consistent with the unique social optimum. Given the realized uncertainty, the socially optimal number of days is unique. However, it is impossible to always make the early and late days both be identical to the socially optimal days. Formally, the ex-post efficient completion time $x_o^A(\theta, \varepsilon)$ must satisfy

$$-\varepsilon \cdot c_1(x_o^A(\theta, \varepsilon), \theta) = c_s. \tag{15}$$

However, due to $d > r$, it cannot be that $d = r = c_s$. Consequently, as implied by (5), both $x^r(\theta, \varepsilon) = x_o^A(\theta, \varepsilon)$ for early completion and $x^d(\theta, \varepsilon) = x_o^A(\theta, \varepsilon)$ for late completion cannot hold simultaneously for all types θ and all uncertainties ε. If the uncertainty were absent, however, the actual working days would equal the bid days. That is, the A+B contract could be ex-post efficient (Lewis and Bajari, 2011).

Second, the A+B contract with uncertainty can be ex-ante efficient. The social welfare in equilibrium is given by

$$W^*(\theta, \varepsilon) = V_c - \varepsilon \cdot c(x^{A^*}(\theta, \varepsilon), \theta) - c_s x^{A^*}(\theta, \varepsilon). \tag{16}$$

As shown in the proof of Proposition 2, if $r < d \leq c_s$, $\partial W^*(\theta, \varepsilon)/\partial \theta < 0$ for any (θ, ε), which implies the ex-ante efficiency of A+B contracts. This may explain the fact that the procurer specifies $r < d \leq c_s$ in practice, because it can guarantee the ex-ante efficiency of A+B contracts in the presence of uncertainty.

Proposition 2. *Under Assumption 1, the A+B contract in the presence of uncertainty is ex-ante efficient if $r < d \leq c_s$, but it cannot be ex-post efficient.*

Proof: See the Appendix.

3. Identification

We consider model identification in an environment for which the data on A+B contracts report the bid pair of cost and working days (P^B, X^B), the bid score S, and the actual working days X^A, where $X^A = X^B$ for on-time completion, $X^A = X^r$ for early completion, and $X^A = X^d$ for late completion.[8] The bid score is observable because the scoring rule is announced by the procurer prior to the bidding stage. We explain how to use these observables and the equilibrium conditions to recover the model primitives $\mathcal{M} = [F_\Theta(\cdot), F(\cdot), c(\cdot,\cdot)]$. Our identification arguments apply conditional on contract characteristics observed in the data. For expositional simplicity, we suppress this dependence whenever there is no ambiguity. A random variable is denoted by an upper-case letter, while realized values are denoted by lower-case letters.

We assume that the cost function is multiplicative in type

$$c(x, \theta) = \theta c_o(x), \tag{17}$$

where $c_o(\cdot)$ is the base cost, which is a function of working days. This specification is used by Krasnokutskaya (2011), which derives the bidder's cost as the product of an individual

[8]We do not use the standard A-only (bid for cost-only) contracts in the identification. Although the choice of standard or A+B contracts seems exogenous because this choice is determined mainly by the observable characteristics of the procurement project such as the engineer's days estimate (Lewis and Bajari, 2011), the model primitives, such as the distribution of bidders' cost types, seem different between standard and A+B contracts. This is because the size of A+B contracts is typically larger than that of standard contracts in terms of working days according to the guidelines from Caltrans. Consequently, the distribution of bidders' cost types associated with A+B contracts seems different from that associated with standard ones (e.g., Marion, 2007; Krasnokutskaya and Seim, 2011).

cost component and an auction-common component in the highway procurement auction. Similar specifications are used in the identification of various economic theories (e.g., Ekeland et al., 2004; Perrigne and Vuong, 2011).

Under the multiplicative specification (17), the model is unidentified without further information because it seems impossible to use the observed bid days to identify both the cost function $c_o(\cdot)$ and the type θ. As shown in the following lemma, an observationally equivalent structure can be obtained by multiplying type θ by some positive constant and dividing $c_o(\cdot)$ by this constant.

Lemma 2. *Consider a structure $\mathcal{M} = [c_o(\cdot), F_\Theta(\cdot), F(\cdot)]$. Define another structure $\widetilde{\mathcal{M}} = [\widetilde{c}_o(\cdot), \widetilde{F}_\Theta(\cdot), F(\cdot)]$, where $\widetilde{c}_o(\cdot) = c_o(\cdot)/\delta$, $\widetilde{F}_{\widetilde{\Theta}}(\cdot) = F_\Theta(\cdot/\delta)$ for some $\delta > 0$. Then, the two structures are observationally equivalent.*

We will study the semi-identification by imposing a parametric specification of the cost function.

Assumption 2.

(a) The cost function is $c(x, \theta) = \theta(\alpha_2 x^2 + \alpha_1 x + \alpha_0)$ with $\alpha_2 > 0$, $\alpha_1 < 0$, and $\alpha_0 \neq 0$.

(b) The lower bound of type support is $\underline{\theta} = 1$.

For the cost function in part (a) of Assumption 2, Assumption 1 implies some restrictions on the cost parameter $\alpha = (\alpha_2, \alpha_1, \alpha_0)$. This parametric cost function is also used for the identifications of the model on wage contracts by D'Haultfœuille and Février (2020) and that on nonlinear pricing with adverse selection by Luo et al. (2018b). Part (b) is a scale normalization implied by Lemma 2. More generally, the normalization of any $\tau \in [0, 1]$-th quantile of the type distribution can be used for identification.

Now, we will explore a quantile relationship between the bid days X^B and the pseudo-type V to identify the cost parameters. Variations in quantiles are explored for the identification of standard auctions (e.g., Guerre et al., 2009). To do so, we use one-to-one mapping between the score S and the pseudo-type V to back out V, and then use the recovered V to identify the cost parameters through quantile relationships. As shown in the proof of Proposition 1, the first-order condition of (7) with respect to score implies

$$s'(v) = (N-1)(s(v)-v)\frac{f_V(v)}{1-F_V(v)} > 0 \tag{18}$$

with boundary condition $s(\overline{v}) = \overline{v}$, where $\overline{v}$ is the upper bound of the pseudo-type, and $F_V(\cdot)$ and $f_V(\cdot)$ are the pseudo-type's cumulative distribution function and density function, respectively. By solving the differential equation (18), we recover the pseudo-type

$$v = s - \frac{1}{N-1}\frac{1-F_S(s)}{f_S(s)}, \tag{19}$$

where $F_S(\cdot)$ and $f_S(\cdot)$ are the score's cumulative distribution function and density function, respectively.

Next, we establish the quantile relationship between the observed X^B and the recovered V to identify the cost parameters. By the definition of pseudo-type, we can rewrite (8) as

$$\begin{aligned} V &= c_u \cdot X^B + \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c\left(X^A(X^B,\theta,\varepsilon),\theta;\alpha\right) + K\left(X^A(X^B,\theta,\varepsilon),X^B,r,d\right)\Big] \qquad (20)\\ &= X^B[c_u - rF(e^r) - d(1-F(e^d))] + \frac{\kappa_0(\alpha_1 X^B + \alpha_2 (X^B)^2) + \alpha_0\kappa_1 m_\varepsilon}{\alpha_1 + 2\alpha_2 X^B} \\ &- \frac{\alpha_1}{2\alpha_2}[rF(e^r) + d(1-F(e^d))] - \frac{\alpha_1^2\kappa_1(m_\varepsilon - \kappa_0/\kappa_1)}{4\alpha_2(\alpha_1+2\alpha_2X^B)} - \frac{(\alpha_1 + 2\alpha_2X^B)\kappa_2}{4\alpha_2\kappa_1} \\ &= g(X^B; c_u, r, d, \alpha, F(\cdot)), \qquad (21) \end{aligned}$$

where $g(\cdot)$ is a nonlinear function known up to the structural primitives α and $F(\cdot)$, $\kappa_2 = r^2\mathbb{E}(\varepsilon^{-1}|\varepsilon \le e^r)F(e^r) + d^2E(\varepsilon^{-1}|\varepsilon \ge e^d)(1-F(e^d))$, $m_\varepsilon = \mathbb{E}(\varepsilon)$ is the mean of the uncertainty, and κ_1 is defined in the following first-order condition of (10) with respect to

x^B

$$c_1(X^B,\theta) = \frac{rF(e^r) + d - dF(e^d) - c_u}{\int_{e^r}^{e^d} \varepsilon dF(\varepsilon)} = \frac{\kappa_0}{\int_{e^r}^{e^d} \varepsilon dF(\varepsilon)} = \kappa_1. \tag{22}$$

Note that $\kappa_0 = rF(e^r) + d - dF(e^d) - c_u$ in (22) is identified, since the probability of early and late completion can be recovered, respectively, as

$$F(e^r) = \Pr(\varepsilon < e^r) = \Pr(X^A < X^B) \text{ and } 1 - F(e^d) = \Pr(\varepsilon > e^d) = \Pr(X^A > X^B). \tag{23}$$

Moreover, the dependence of V only on X^B in $g(\cdot)$ follows the fact that θ in $X^A(X^B,\theta,\varepsilon)$ of (20) can be expressed in terms of X^B, where $X^A(X^B,\theta,\varepsilon)$ below is obtained by using (11) in Proposition 1

$$X^A(X^B,\theta,\varepsilon) = \begin{cases} X^r = (-r\varepsilon^{-1}\theta^{-1} - \alpha_1)(2\alpha_2)^{-1} & \textit{if } \ \varepsilon \le e^r, \\ X^B = (\kappa_1\theta^{-1} - \alpha_1)(2\alpha_2)^{-1} & \textit{if } \ \varepsilon \in [e^r, e^d], \\ X^d = (-d\varepsilon^{-1}\theta^{-1} - \alpha_1)(2\alpha_2)^{-1} & \textit{if } \ \varepsilon \ge e^d. \end{cases} \tag{24}$$

As (24) shows, the ex-post uncertainty ε affects the early working days X^r or the late workings days X^d in the construction stage, and the number of bid days in the bidding stage does not depend on the ex-post uncertainty.

Note that $dv(\theta)/d\theta > 0$ and $dX^B(\theta)/d\theta > 0$ in equilibrium. Therefore, we substitute $Q_V(\tau)$ and $Q_{X^B}(\tau)$, respectively, for V and X^B in (21), where $Q_V(\tau)$ and $Q_{X^B}(\tau)$ are the respective quantiles of V and X^B for any $\tau \in [0,1]$. As a result, we obtain

$$Q_V(\tau) = g(Q_{X^B}(\tau); c_u, r, d, \alpha, F(\cdot)). \tag{25}$$

After algebraic rearrangements, (25) becomes

$$(26) \qquad 0 = \beta_0 + \beta_1 Q_V(\tau) + \beta_2 Q_V(\tau) Q_{X^B}(\tau) + \beta_3 Q_{X^B}(\tau) + \beta_4 (Q_{X^B}(\tau))^2,$$

where $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$ is given by

$$\beta_0 = \alpha_0 \frac{m_\varepsilon \beta_1^3}{2\beta_2 \alpha_1^3} + \frac{\beta_1^2}{\beta_2} - \frac{\beta_1^2 (m_\varepsilon \beta_1^2 + 2\kappa_0 \alpha_1^2 \beta_2)}{4\alpha_1^2 \beta_2^2} - r\frac{\beta_1^2}{2\beta_2} \frac{\beta_1 + \beta_2 \mu_r}{\beta_1 + \beta_2 \mu_B} F(e^r) - d\frac{\beta_1^2}{2\beta_2} \frac{\beta_1 + \beta_2 \mu_d}{\beta_1 + \beta_2 \mu_B} (1 - F(e^d)),$$

$$\begin{aligned} \beta_1 &= -4\kappa_1 \alpha_1 \alpha_2, \\ \beta_2 &= -8\kappa_1 \alpha_2^2, \\ \beta_3 &= -\beta_1 (c_u + \kappa_0) + 2\beta_1 \left[rF(e^r) + d(1 - F(e^d)) \right] - 2\alpha_1^2 \beta_2 \kappa_2 / \beta_1, \\ \beta_4 &= -\beta_2 \left\{ c_u - \left[rF(e^r) + d(1 - F(e^d)) \right] \right\} - \kappa_0 \beta_2 / 2 - \alpha_1^2 \beta_2^2 \kappa_2 / \beta_1^2 \end{aligned}$$

with $\mu_r = \mathbb{E}(X^r)$, $\mu_d = \mathbb{E}(X^d)$, and $\mu_B = \mathbb{E}(X^B)$. Due to the nonlinear relationship between X^B and V in (21), the full rank property holds in (26) in general. Therefore, $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$ is identified by choosing any five different values of $\tau \in (0, 1)$ to construct five linearly independent equations. For the remaining identification, we provide an intuitive explanation and leave the details to the Appendix. Since β is a system of equations of $(\alpha_0, \alpha_1, \alpha_2, m_\varepsilon)$, we can recover the slope parameters (α_1, α_2) by combining the normalizations in Assumption 2.

We use the bid days X^B to back out the bidder's type θ and recover the type distribution $F_\Theta(\cdot)$ on its support $\mathcal{S}_\Theta$. Since the intercept parameter α_0 is involved only in β_0, and β_0 includes the mean uncertainty m_ε, α_0 is identified if m_ε is known. Note that the bid cost P^B has no additional identification power because it is simply a known linear transformation of the bid score and bid days.[9] Consequently, one may not need to use

[9]As shown in the proof of Proposition 3, the bid cost can be written as a known function of the

the bid cost in estimation.

Finally, we use the early or late working days to recover the corresponding uncertainty. For contracts finished on time, the actual days equals the bid days, which does not rely on uncertainty. Consequently, we cannot recover the uncertainty on its entire support, and can identify the truncated distribution of ε, denoted by $G(\cdot)$, on the support $\widetilde{\mathcal{S}}_\varepsilon = \mathcal{S}_r \cup \mathcal{S}_d \subset \mathcal{S}_\varepsilon$ with $\mathcal{S}_r = \{\varepsilon : \varepsilon \leq \varepsilon^r\}$ and $\mathcal{S}_d = \{\varepsilon : \varepsilon \geq \varepsilon^d\}$. The uncertainty distribution $F(\cdot)$ is identified on $\widetilde{\mathcal{S}}_\varepsilon$ as

$$F(\varepsilon) = G(\varepsilon)F(\varepsilon^r) \text{ if } \varepsilon \in \mathcal{S}_r, \text{ and } F(\varepsilon) = G(\varepsilon)(1 - F(\varepsilon^d)) \text{ if } \varepsilon \in \mathcal{S}_d. \tag{27}$$

Proposition 3. *Suppose that Assumptions 1-2 hold, and the mean uncertainty is known. Then, the cost parameter $\alpha = (\alpha_2, \alpha_1, \alpha_0)$ is identified, and the type distribution $F_\Theta(\cdot)$ and the uncertainty distribution $F(\cdot)$ are identified on the supports $\mathcal{S}_\Theta$ and $\widetilde{\mathcal{S}}_\varepsilon$, respectively.*

An immediate result of Proposition 3 is that if the uncertainty distribution is parameterized, with, say, the commonly used log normal distribution, the above-recovered uncertainty can be used to identify the parameters of the uncertainty distribution.

Corollary 1. *Suppose that Assumptions 1-2 hold and the uncertainty distribution is parameterized. Then, the cost parameter $\alpha = (\alpha_2, \alpha_1, \alpha_0)$ and parameters of the uncertainty distribution are identified, and the type distribution $F_\Theta(\cdot)$ is identified on the support $\mathcal{S}_\Theta$.*

4. A+B Contracts: Background and Data

Caltrans is a government department of the state of California that is responsible for the planning, construction and maintenance of public transportation facilities such as highways, bridges, and railways. The A+B contract design was introduced by Caltrans in

observables and those model primitives already identified.

the 1990s as an experiment for emergency-type projects following criticism that highway construction took too much time, and it was extended to non-emergency-type projects in 2000. First, the engineer estimates the project's cost and a target number of working days needed for project completion. The maximum number of lanes that can be closed during each phase of the project and their closure times are also specified by engineers. Once informed of the engineer's estimates, scoring rule, and time incentives, contractors draw private costs for completing the project and quote their costs and completion time in the bidding stage.[10] The contract is awarded to the contractor with the lowest score according to the announced scoring rule. In the construction stage, faced with realized uncertainty, the contractor may have actual working days that differs from its bid days. This strategic deviation is the contractor's trade-off between the cost increase from early completion and the corresponding bonus or between the cost reduction from late completion and the corresponding penalty.

4.1. Data

The data contain 1284 bids submitted by contractors in 223 A+B contracts from 2003 to 2018 in California. We use the same source as Lewis and Bajari (2011) and extend their data covering the period from 2003 to 2008.[11] These contracts include barrier construction, bridge repair or resurfacing, new lane and ramp construction, road rehabilitation, slope work and widening/realignment. For each contract, the data report the bid pair of cost and completion time submitted by each bidder, the number of bidders, the engineer's estimated project cost and working days, daily incentives and disincentives, user cost as specified in the scoring rule, and the actual working days.

[10] Caltrans sets the user cost in the scoring rule based on a standardized statewide formula for liquidated damages, which depends on the engineer's estimates, type of work, and the expenses of the resident engineer's office, among other factors.

[11] The data were obtained at http://www.dot.ca.gov/.

We have information on the daily commuter cost, which is a measure of the negative externality of project construction to commuters.[12] In addition, we have additional characteristics in the data, including each contractor's capacity, measured as the total value of all contracts held by a particular contractor during our sample period, the distance between a contractor's location and the project work site, and an indicator for whether the contract is federally funded.

Table 1 presents the summary statistics of the data at the contract and bidder levels in Panels A and B, respectively. In Panel A, the average estimated cost and completion time are \$22.24 million and 318 days, respectively. The average value of the user cost, \$11,980, lies between the daily incentive, averaged at \$7,190, and the daily disincentive, averaged at \$13,560. The user cost is approximately one-quarter of the daily commuter cost, which is \$52,130, on average. The average winning bid for the cost is \$18.65 million, which is slightly smaller than the engineer's average estimated cost. The number of bidders ranges from 2 to 14, with an average of 6. Most contracts are funded by the federal government. On average, the firm's capacity is \$45.40 million, and its distance to the work site is 77.99 miles. In addition, we report the actual working days, the contract days, and the difference between them.[13] The average contract days and actual working days are 231 and 267, respectively. On average, an A+B contract is delayed by approximately 36 days, which suggests the relevance of post-auction uncertainty in A+B contracts. In Panel B, the average bid cost is \$20.5 million, and the average bid days is 199 days. The bid cost and bid days are smaller than the engineer's estimated cost and days, respectively. Since the bid score is a weighted sum of bid cost and bid days, the average bid score (\$23.3 million) is smaller than the average engineer's estimated score (\$26.83 million).

[12]The commuter cost is constructed by following Lewis and Bajari (2011). Using the information on traffic volumes, the percentage of trucks at each of the contract locations, the time value and the delay, we calculate the daily commuter cost by multiplying the traffic, the delay, and the time value.

[13]For each contract, the contract days is the number of days bid by the winning contractor.

[Insert Table 1 here]

Table 1: Summary Statistics for A+B Highway Construction Contracts in California

Variable	Mean	Std. Dev.	Min	10-th pctile	Median	90-th pctile	Max
Panel A: Contract Level							
Engineer Cost ($M)	22.24	31.19	0.50	1.93	11.35	55.44	274.67
Engineer Days	317.99	229.17	45.00	120.00	245.00	630.00	1370.00
Usercost ($K)	11.98	10.06	1.80	4.20	10.50	20.72	93.99
Incentive Payments ($K)	7.19	6.04	1.08	2.52	6.30	12.43	56.39
Liquidated Damages ($K)	13.56	16.66	0.30	4.30	10.50	21.00	190.50
Engineer Score ($M)	26.83	36.80	0.88	2.51	14.29	65.10	316.69
Winning Bid Cost ($M)	18.65	26.30	0.33	1.66	9.81	44.87	214.81
Number of Bidders	5.81	2.33	2.00	3.00	6.00	9.00	14.00
Federal Contract	0.77	0.42	0.00	0.00	1.00	1.00	1.00
Firm Capacity ($M)	45.40	36.93	0.70	5.21	37.16	85.69	214.81
Distance (miles)	77.99	129.95	1.55	8.01	28.57	290.86	802.14
Commuter Cost ($K)	52.13	50.83	0.11	3.75	33.05	129.35	213.23
Contract Days	230.96	225.60	25.00	50.00	150.00	510.00	1140.00
Working Days	266.81	286.53	30.00	60.00	159.00	591.00	1855.00
Working Days-Contract Days	35.85	122.98	-281.00	-23.00	0.00	156.00	750.00
Panel B: Bidder Level							
Bid Cost ($M)	20.50	28.66	0.33	2.22	10.30	49.43	445.28
Bid Days	199.17	157.97	20.00	65.00	150.00	381.00	1550
Bid Score ($M)	23.30	32.55	0.41	2.62	11.93	54.01	504.49
Firm Capacity ($M)	35.30	34.96	0.00	0.00	26.86	85.69	214.80
Distance (miles)	83.57	165.39	0.15	10.25	32.80	231.34	2517.18

Note: Among all 223 A+B contracts, 205 contracts have been completed. Panel A is the summary statistics at the contract level with 205 contracts completed. Panel B is the summary statistics at the bidder level with 1284 bids in 223 contracts. The contract days in Panel A is the number of days bid by the winning contractor.

4.2. Motivating evidence

We present some preliminary evidence that motivates our model. Figure 1 presents histograms of the difference and ratio between contract days and actual working days. We find that more than two-thirds of contracts are not completed on time. Among the 205 completed A+B contracts, 30 percent (61 cases) were completed on time, 24 percent (49 cases) were completed earlier than bid, and 46 percent (95 cases) were completed late. This provides preliminary evidence that contractors may adjust the completion time by deviating from the bid days in response to unexpected construction uncertainty.

[Insert Figure 1 here]

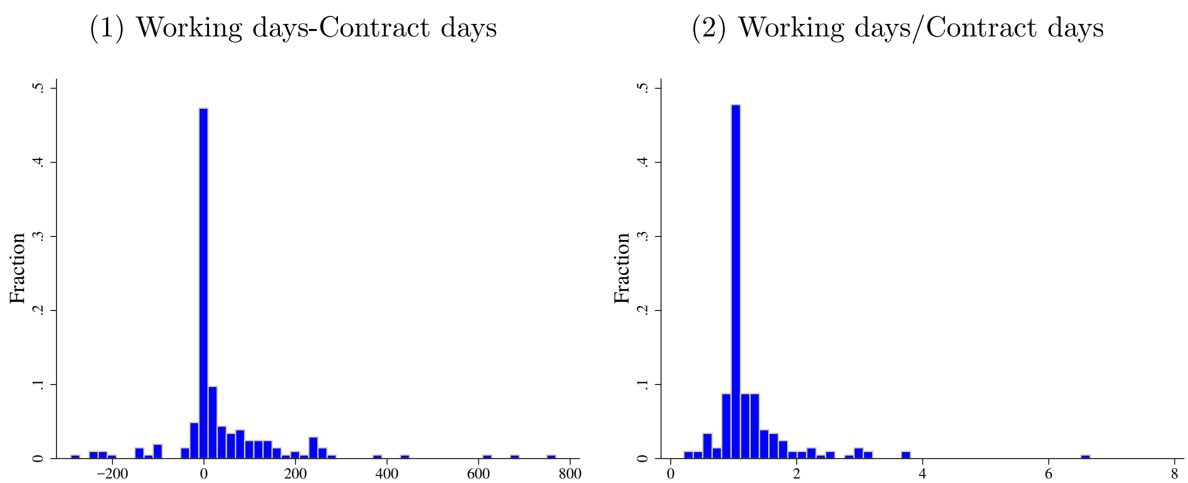

Figure 1: Comparisons between working days and contract days
Note: Figure 1 shows the histograms of the difference and ratio between actual working days and contract days in the data, where the contract days is the number of days bid by the winning contractor.

Moreover, Figure 2 plots the empirical distribution of the deviation in days for contracts that are finished early or late, where the early (late) deviation days is the difference between the early (late) working days and the bid days. As implied by Figure 2, for an A+B contract with an average engineer's completion time of 318 days, there is a 50 percent chance that the working time saved (delayed) will be greater than 13 days (65 days).

These results suggest that the ex-post uncertainty, to some extent, leads to contractor's strategic deviation of their actual days from the bid days.

[Insert Figure 2 here]

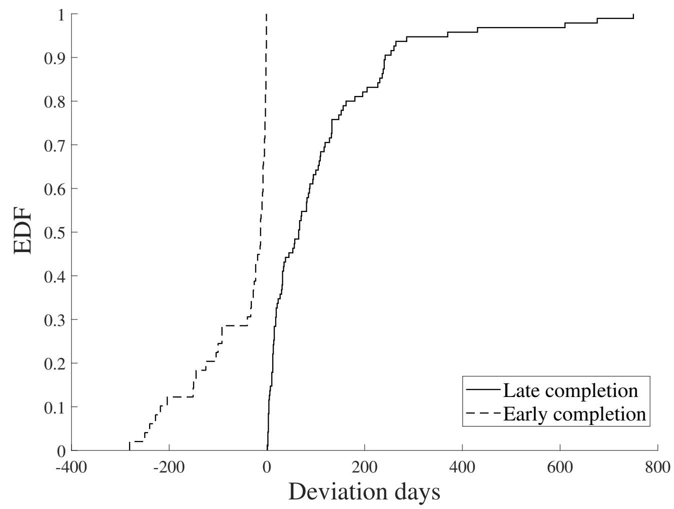

Figure 2: Empirical distribution functions of deviation days

Note: Figure 2 plots the empirical distribution functions of the deviation days for early and late completions, where the early (late) deviation days = early (late) working days - bid days. Hence, the upper (lower) bound of the support for the early (late) deviation days is zero.

5. Empirical Analysis

In this section, we follow the identification strategy to estimate a semiparametric model that accounts for the heterogeneity of contracts in the data.

5.1. Estimation strategy

First, due to the limited sample size for early or late working days, we use linear regressions to analyze the effects of various characteristics on the early or late working days,

$$\mu_r(z) = \mathbb{E}(X^r|Z = z) = z'\chi^r \text{ and } \mu_d(z) = \mathbb{E}(X^d|Z = z) = z'\chi^d, \tag{28}$$

where χ^r and χ^d are unknown parameters, and Z is the engineer's estimated days, which can capture project heterogeneity.[14] We consider various linear regressions of early and late working days using different characteristics. As indicated in Table 2 below, only the engineer's days has statistically significant effects on the actual working days in all specifications. Therefore, as in Krasnokutskaya and Seim (2011), we use the auction-level characteristics (here, the engineer's days) in the remaining empirical analyses.

In the linear regression specifications for early or late working days presented above, there are no clear implications for the relationship between bid days and early or late working days. To circumvent potential conflicts in the specifications and because the bid days has a larger sample size available than early or late working days, we use nonparametric procedures to estimate the conditional mean of bid days,

$$\mu_B(z) = \mathbb{E}(X^B|Z = z) = g(z), \tag{29}$$

where $g(\cdot)$ is an unknown function. The corresponding estimators are $\widehat{\chi}^r$, $\widehat{\chi}^d$, and $\widehat{g}(\cdot)$.

We follow (19) to estimate the pseudo-type V. Using the traditional nonparametric kernel method, we estimate the conditional distribution of score $F_{S|S^E}(s|s^E)$ and its density $f_{S|S^E}(s|s^E)$ given $S^E = s^E$,[15], where $s^E \equiv p^E + c_u x^E$ is the engineer's estimated score, with p^E and x^E being the engineer's estimated cost and days, respectively. As a result,

[14]One might want to include the engineer's estimated cost in Z to control for auction-specific heterogeneity. Since the dependent variable in the regression (28) is about the working days, it seems natural to use the days-related variable in the regression. Moreover, the engineer's days and cost are highly positively correlated, and the adjusted R^2 in Table 2 increases a little after we include engineer's cost. Therefore, we do not include engineer's cost in Z.

[15]For details, see Li and Racine (2007).

the estimate of bidder i's pseudo-type in project j is given by

$$\widehat{v}_{ji} = s_{ji} - \frac{1}{n_j - 1} \frac{1 - \widehat{F}_{S|S^E}(s_{ji}|s_j^E)}{\widehat{f}_{S|S^E}(s_{ji}|s_j^E)}, \tag{30}$$

where n_j is the number of bidders, and s_{ji} and s_j^E are bidder i's bid score and the engineer's score, respectively.[16]

Next, we consider the parametric estimation of the uncertainty distribution. Suppose that the conditional distribution of ε given $Z = z$ is lognormal $(\mu, \sigma^2(z))$ with $\sigma(z) = z'\psi_\sigma$, where (μ, ψ_σ) are unknown parameters. To construct a tractable likelihood function for (μ, ψ_σ), we specify the relationship between the cutoff levels of uncertainty and characteristics as

$$e^d(z) = exp(z'\psi_d), \tag{31}$$

where ψ_d is an unknown parameter.[17] Combining this with (4) implies $e^r(z) = rd^{-1}exp(z'\psi_d)$. Let I^R be the indicator for early completion, where $I^R = 1$ for early completion, and $I^R = 0$ otherwise. Let I^D be the indicator for late completion, where $I^D = 1$ for late com-

[16]We follow Bajari et al. (2014) and An and Tang (2019) to estimate a separate conditional distribution of the bid score for contracts where the number of bidders is $n = 2, 3$ (107 bids), $n = 4$ (147 bids), $n = 5$ (164 bids), $n = 6$ (203 bids), $n = 7$ (224 bids), $n = 8$ (191 bids) and $n \geq 9$ (248 bids). These estimates seem quite precise compared to the ideal case without any grouping. This is because the majority (72 percent in our data) of the bid scores are used in correctly specified models. Moreover, we choose $n = 3$ in the estimation for the group $n = 2, 3$ given that contracts with $n = 3$ have 87 bids, and choose $n = 10$ in the estimation for the group $n \geq 9$ given that contracts with $n = 10$ have 120 bids.

[17]Given the complexity of the multistep estimation and the nonparametric specifications of bid days and bid score, an extra nonparametric regression of cutoff levels on characteristics, which is a highly nonlinear and nonparametric estimation due to $e^d(z)$ entering $\Phi(\cdot)$ in (33), would generate less accurate estimates given the relatively limited sample size. Therefore, we use the parametric specifications of the cutoff levels only to maintain the tractability of the estimation.

pletion, and $I^D = 0$ otherwise. $I^B = 1 - I^R - I^D$ is the indicator for on-time completion. As a result, we obtain

$$\mathbb{E}(I^R|Z = z) = F(e^r(z)|z) \text{ and } \mathbb{E}(I^D|Z = z) = 1 - F(e^d(z)|z), \tag{32}$$

which implies $\mathbb{E}(I^B|Z = z) = F(e^d(z)|z) - F(e^r(z)|z)$. The likelihood function of $(\mu, \psi_\sigma, \psi_d)$ is given by

$$\mathcal{L}(\mu, \psi_\sigma, \psi_d) = \prod_{j=1}^{J} \left[F(e^r(z_j)|z_j)\right]^{I_j^R} \left[1 - F(e^d(z_j)|z_j)\right]^{I_j^D} \left[F(e^d(z_j)|z_j) - F(e^r(z_j)|z_j)\right]^{I_j^B}, \tag{33}$$

where

$$\begin{aligned} F(e^r(z_j)|z_j) &= \Pr(\varepsilon \le e^r(z_j)|z_j) = \Pr(log(\varepsilon) \le log(e^r(z_j))|z_j) = \Phi\left(\frac{log(r_j/d_j) + z_j\psi_d - \mu}{z_j\psi_\sigma}\right), \\ F(e^d(z_j)|z_j) &= \Pr(\varepsilon \le e^d(z_j)|z_j) = \Pr(log(\varepsilon) \le log(e^d(z_j))|z_j) = \Phi\left(\frac{z_j\psi_d - \mu}{z_j\psi_\sigma}\right). \end{aligned}$$

Using MLE allows us to obtain the corresponding estimators $(\widehat{\mu}, \widehat{\psi}_d, \widehat{\psi}_\sigma)$.

Finally, based on a heterogeneous quantile relationship implied by (26), we construct the set of moments to estimate cost parameters by using an extremum estimator

$$(\widehat{\alpha}_1, \widehat{\alpha}_2) = \underset{(\alpha_1,\alpha_2)}{\operatorname{argmin}}\, \mathcal{M}_J(\alpha_1, \alpha_2), \tag{34}$$

where

$$\mathcal{M}_J(\alpha_1, \alpha_2) = \frac{1}{J}\sum_{j=1}^{J}\left\{\frac{1}{n_j}\sum_{i=1}^{n_j}\left[\beta_0(z_j) + \beta_1(z_j)\widehat{v}_{ji} + \beta_2(z_j)\widehat{v}_{ji}x_{ji}^B + \beta_3(z_j)x_{ji}^B + \beta_4(z_j)(x_{ji}^B)^2\right]^2\right\},$$

and the definitions of $\{\beta_k(z_j)\}_{k=0}^4$ are stated in the Appendix. For the identification of (α_1, α_2) in the presence of heterogeneity, we normalize the cost intercept $\alpha_0 = 1$.

Combining the above estimates, we obtain the type estimate for bidder i in contract j as $\widehat{\theta}_{ji} = \widehat{\kappa}_1(z_j)(\widehat{\alpha}_1 + 2\widehat{\alpha}_2 X^B_{ji})^{-1}$, where $\widehat{\kappa}_1(z_j)$ is defined in the Appendix. The estimates of $\{(\widehat{\theta}_{ji})^{n_j}_{i=1}\}^J_{j=1}$ can be used to obtain the estimator of conditional type distribution $F_{\Theta|Z}(\cdot|z)$ given $Z = z$.

5.2. Empirical results

Table 2 reports the estimation results for separate regressions of early and late working days on different characteristics. We find that only the engineer's estimated days has a significant positive effect on the actual working days in all specifications. This result implies that projects with more engineer's days tend to need more time to be finished, which is obvious because engineer's days reflects the scale and complexity of the construction process. The effects of a one day increase in engineer's days on the early and delayed working days are 0.752 and 1.023, respectively. However, the distance to the work site, the contractor's capacity, and whether the contract is funded by the federal government have no significant effects on working days.[18]

[Insert Table 2 here]

[18]The results for the significance of only engineer's days remain unchanged if we use the logarithm of capacity and distance in the linear regressions of early or late days.

Table 2: Early and Late Working Days Regression

	Early Working Days				Late Working Days			
Engineer Days	0.752***	0.730***	0.750***	0.752***	1.023***	0.892***	0.889***	0.885***
	(0.063)	(0.088)	(0.074)	(0.072)	(0.125)	(0.116)	(0.12)	(0.121)
Capacity		2.177×10^{-7}	2.112×10^{-7}	2.322×10^{-7}		1.932×10^{-6}	2.011×10^{-6}	2.019×10^{-6}
		(5.987×10^{-7})	(5.823×10^{-7})	(6.035×10^{-7})		(8.978×10^{-7})	(9.779×10^{-7})	(9.878×10^{-7})
Distance			0.207	0.227			-0.056	-0.052
			(0.208)	(0.216)			(0.191)	(0.190)
Federal				-23.451				-10.643
				(28.401)				(62.283)
Constant	-19.502	-21.296	-39.005	-22.461	-17.141	-64.042	-61.843	-53.030
	(25.773)	(26.742)	(32.400)	(27.864)	(41.241)	(44.176)	(47.927)	(71.456)
Observations	49	49	49	49	95	95	95	95
Adjusted R^2	0.623	0.617	0.617	0.611	0.581	0.608	0.601	0.596

Note: Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

To examine the goodness of fit of our estimates, we compare the fitted bid days with the fitted early and late working days. Figure 3 indicates a clear pattern for the vast majority (83 percent) of the results, in which the fitted bid days is larger than the fitted early days and smaller than the fitted late days; this is consistent with the model prediction that the bid days is larger (smaller) than the early (late) days.

[Insert Figure 3 here]

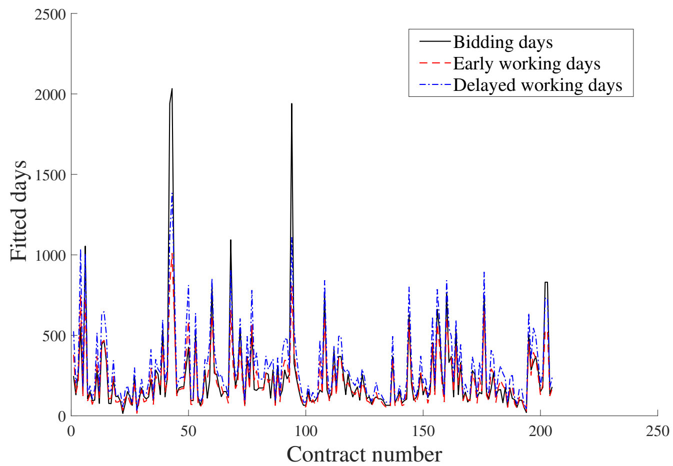

Figure 3: Fitted bid days and actual working days

Note: Figure 3 plots the fitted bid days, early working days and late working days based on $\widehat{\mu}_B(z)$, $\widehat{\mu}_r(z)$, and $\widehat{\mu}_d(z)$, respectively.

Table 3 reports the estimates of the parameters of the uncertainty distribution, the cutoff uncertainty regression and the cost parameters. All estimates are statistically significant. The estimates of the mean and the standard deviation for $\log(\varepsilon)$ are -0.069 and 0.004, respectively. The parameter estimate in the cutoff uncertainty regression is 0.0002. The cost parameters α_1 and α_2 are significantly negative and positive, respectively, which is consistent with Assumption 2.

Although the estimates of cost parameters (α_1, α_2) are tiny, the estimate of the marginal effect of working days x on the cost c, $\theta(\alpha_1 + 2\alpha_2 x)$, is of magnitude 10^4 on average.[19] This result is consistent with the fact that the average ratio of engineer's cost to engineer's days is also of magnitude 10^4. Moreover, the estimated marginal effects of working days are very close under different levels of normalization for α_0 because the estimates of both cost parameters and bidders' types vary with the normalization of α_0.

[Insert Table 3 here]

Table 3: Estimates of Other Parameters

	Parameters/Variables	Estimates
Distribution of Uncertainty	Mean of log(Uncertainty)	-0.069**
		(0.031)
	SD of log(Uncertainty)	0.004***
		(4.444×10^{-4})
Cutoff Uncertainty	Engineer Days	1.783×10^{-4}**
		(8.832×10^{-5})
Cost Parameters	Working Days	-1.755×10^{-4}***
		(1.203×10^{-5})
	Working Days^2	7.702×10^{-9}***
		(9.156×10^{-10})

Note: Bootstrap standard errors in parentheses are calculated using 500 bootstrap samples. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Finally, Figure 4 illustrates the estimated distribution function for the bidder's type θ conditional on the lower, middle and upper quartiles of engineer's days x^E. As stated above, the estimates of types are reasonable in the sense that the magnitude of the mean is 10^8, which implies that the estimated marginal effect of contractor's working days is of the same magnitude as the average effect of engineer's estimated days.

[19] This result is obtained by combining the results that the estimates of bidder's type θ have a magnitude 10^8 on average, the working days is at most of magnitude 10^3, and the magnitudes of the α_1 and α_2 estimates are 10^{-4} and 10^{-9}, respectively.

[Insert Figure 4 here]

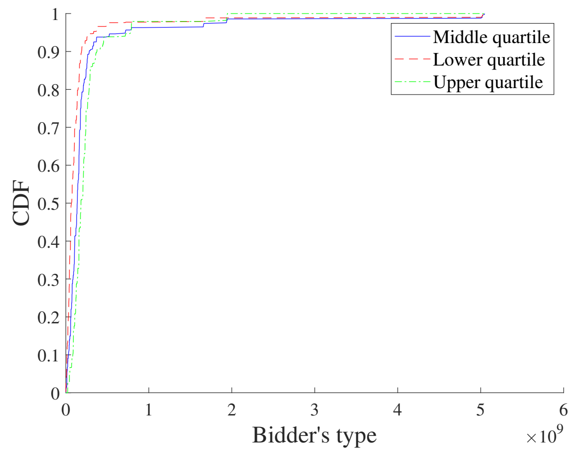

Figure 4: Estimate of conditional distribution of bidder's type

Note: Figure 4 plots the estimate of the conditional distribution of bidder's type θ given lower, middle and upper quartiles of engineer's estimated days x^E.

6. Counterfactuals

In this section, we first investigate whether more flexible time incentive schemes could allow for some optimization within the class of A+B contracts by analyzing the efficiency of A+B contracts in a case where the time incentives impose a larger penalty for late completion. Next, we compare the welfare performance between A+B contracts and lane rental contracts. Lane rental contracts, which were introduced by the United Kingdom, perform well in reducing completion time in practice. One common practical motivation for A+B and lane rental contracts is to induce contractors to internalize the negative construction externality and reduce construction time in heavily populated areas or on busy roads (Srinivasan and Harris, 1991; Herbsman and Glagola, 1998). However, there have been heated disputes over which contract mechanism is preferable (Strong, 2006). To resolve these policy disputes, we will compare the efficiency between A+B and lane rental contracts in the presence of uncertainty. Then, we use the estimates from Section 5 to quantify the differences in various costs between different contracts.

6.1. Alternative time incentives in A+B contracting

We consider two alternative time incentives with a larger penalty for late completion than the penalty used in the data. The first alternative time incentives $K_Q(\cdot)$ impose a quadratic penalty for late completion

$$K_Q(x^A, x^B, r, d, \delta) = \mathbb{1}(x^A \leq x^B) \cdot r \cdot (x^A - x^B) + \mathbb{1}(x^A \geq x^B) \cdot d \cdot \left[(x^A - x^B) + \frac{\delta \cdot (x^A - x^B)^2}{2}\right], \tag{35}$$

where $\delta > 0$ is the parameter determining the additional penalty aside from the linear part. As long as the project is finished late, the penalty under $K_Q(x^A, x^B, r, d, \delta)$ is larger than that under $K(x^A, x^B, r, d)$. Note that $K_Q(x^A, x^B, r, d, 0) = K(x^A, x^B, r, d)$.

The second alternative time incentives $K_P(\cdot)$ impose a piecewise linear penalty for late completion, that is, they include a larger penalty for a more serious completion delay

$$\begin{aligned} K_P(x^A, x^B, r, d, \widetilde{d}, \rho) =& \mathbb{1}(x^A \leq x^B) \cdot r \cdot (x^A - x^B) + \mathbb{1}(x^B \leq x^A \leq \rho x^B) \cdot d \cdot (x^A - x^B) \\ &+ \mathbb{1}(x^A \geq \rho x^B) \cdot \left[\widetilde{d} \cdot (x^A - \rho x^B) + d \cdot (\rho - 1) x^B\right], \end{aligned} \tag{36}$$

where $r < d < \widetilde{d}$ and $\rho > 1$. The two parameters ρ and $\widetilde{d}$ respectively specify the criterion for more seriously delayed completion and the corresponding larger penalty. If the delay is mild in terms of $x^A \leq \rho x^B$, the penalty under $K_P(\cdot)$ is the same as that in $K(\cdot)$; if the delay is severe in terms of $x^A > \rho x^B$, the delayed days net the mild delayed days, $x^A - \rho x^B$, is penalized more heavily due to $\widetilde{d} > d$. Note that $K_P(x^A, x^B, r, d, d, 1) = K(x^A, x^B, r, d)$.

Corollary 2. *Under Assumption 1 and the time incentives either $K_Q(\cdot)$ or $K_P(\cdot)$, the*

A+B contract in the presence of uncertainty is not necessarily ex-ante efficient if $r < d \leq c_s$*, and it cannot be ex-post efficient.*

Proof: See the Appendix.

If $r < d \leq c_s$, A+B contracts under $K(\cdot)$ must be ex-ante efficient in Proposition 2. However, they are not necessarily ex-ante efficient under $K_Q(\cdot)$ or $K_P(\cdot)$ according to Corollary 2. Intuitively, a more flexible time incentive includes more parameters, requiring more constraints for the ex-ante efficiency. As the proof in the Appendix suggests, it seems technically difficult for CalTrans to specify r and d to obtain the ex-ante efficient A+B contracts under $K_Q(\cdot)$ or $K_P(\cdot)$. Accordingly, the A+B contract under $K(\cdot)$ seems preferable because it is more likely to be ex-ante efficient than one under $K_Q(\cdot)$ or $K_P(\cdot)$.

In addition, as in Proposition 2, the ex-post inefficiency of A+B contracts under $K_Q(\cdot)$ or $K_P(\cdot)$ in Corollary 2 also results from the fact that the two distinct time incentives for early and late completion lead to that the contractor's actual working days cannot always equal the unique socially optimal number of working days. It seems more interesting to investigate whether another contracting mechanism exists that is efficient in the presence of ex-post uncertainty.

6.2. Efficiency of lane rental contracts

Given the ex-post inefficiency of A+B contracts in the above analyses, we examine the ex-post efficiency of an alternative contract, a lane rental contract, in the presence of ex-post uncertainty. In the lane rental contract, each bidder quotes a cost bid, and the bidder with the lowest bid wins the contract. There is no required completion date; instead, the winning contractor needs to pay a daily fixed amount $d_L > 0$ for the lanes occupied throughout the construction stage. Recall that the ex-post inefficiency of A+B contracts with time incentives in the presence of ex-post uncertainty arises from two distinct time incentives for early and late completion. In contrast, the lane rental contract can be ex-post efficient if the lane rental fee d_L, which alone determines the incentive of the lane rental contract, is chosen appropriately.

Under lane rental contracts, the contractor's incentive cost with working days x^A is given by

$$K_L(x^A, d_L) = d_L \cdot x^A. \tag{37}$$

The equilibrium number of working days under uncertainty ε is $x_L^{A^*}(\theta, \varepsilon) = x^{d_L}(\theta, \varepsilon)$ with

$$-\varepsilon \cdot c_1(x^{d_L}(\theta, \varepsilon), \theta) = d_L, \tag{38}$$

which implies that lane rental contracts are ex-post efficient in the presence of uncertainty when $d_L = c_s$.

Lane rental contracts are ex-ante efficient if $d_L \leq c_s$. The social welfare of lane rental contracts in equilibrium for any (θ, ε) is given by

$$W_L^*(\theta, \varepsilon) = V_c - \varepsilon \cdot c(x^{d_L}(\theta, \varepsilon), \theta) - c_s x^{d_L}(\theta, \varepsilon)$$

with

$$\partial W_L^*(\theta, \varepsilon)/\partial\theta = (d_L - c_s)\partial x^{d_L}(\theta, \varepsilon)/\partial\theta - \varepsilon \cdot c_2(x^{d_L}(\theta, \varepsilon), \theta).$$

It can be shown that $\partial x^{d_L}(\theta, \varepsilon)/\partial\theta > 0$ by using (38). When $d_L \leq c_s$, it follows that $\partial W_L^*(\theta, \varepsilon)/\partial\theta < 0$ by combining $c_2(\cdot, \cdot) > 0$. Therefore, lane rental contracts with uncertainty are ex-ante efficient if $d_L \leq c_s$. In addition, the pseudo-type in the lane rental contract is

$$v_L(\theta) = \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(x^{d_L}(\theta, \varepsilon), \theta) + d_L \cdot x^{d_L}(\theta, \varepsilon)\Big] \tag{39}$$

and

$$v_L'(\theta) = \mathbb{E}_\varepsilon \left[\varepsilon \cdot c_2(x^{d_L}(\theta,\varepsilon),\theta)\right] > 0.$$

Using arguments similar to those applied to A+B contracts, we obtain the equilibrium bid and the efficiency of the lane rental contract.

Proposition 4. *Under Assumption 1, for the lane rental contract in the presence of uncertainty, we obtain the following results.*

(i) The equilibrium number of working days is

$$x_L^{A^*}(\theta,\varepsilon) = x^{d_L}(\theta,\varepsilon) \tag{40}$$

with

$$-\varepsilon \cdot c_1(x^{d_L}(\theta,\varepsilon),\theta) = d_L. \tag{41}$$

(ii) The equilibrium cost bid is

(42)

$$p_L^*(\theta) = \mathbb{E}_\varepsilon\left[\varepsilon \cdot c(x^{d_L}(\theta,\varepsilon),\theta) + d_L \cdot x^{d_L}(\theta,\varepsilon)\right] + \int_\theta^{\overline{\theta}} \mathbb{E}_\varepsilon[\varepsilon \cdot c_2(x^{d_L}(\theta,\varepsilon),\theta)]\left[\frac{1-F_\Theta(t)}{1-F_\Theta(\theta)}\right]^{N-1} dt.$$

(iii) The lane rental contract is ex-post efficient if $d_L = c_s$ and ex-ante efficient if $d_L \leq c_s$.

According to Proposition 2 and 4, the sufficient conditions for the ex-ante efficiency of A+B and lane rental contracts are very similar in that the daily disincentive is smaller than or equal to the daily commuter cost. However, Proposition 4 suggests that lane rental contracts should be preferred, since A+B contracts are not ex-post efficient. For lane rental contracts, the daily penalty d_L is usually equal to the daily commuter cost c_s in practice (Herbsman and Glagola, 1998). This result is consistent with our sufficient conditions for ex-ante and ex-post efficiency in Proposition 4.

6.3. Counterfactual results

In this subsection, we conduct counterfactual analysis to compare the various costs between A+B and lane rental contracts by using the model estimates. The counterfactual procedure consists of four steps. First, we use the observed bid days in all contracts to obtain the estimator $\widehat{F}_{B|Z}(b|z)$ of the conditional distribution of bid days given $Z = z$. Second, for each observation of A+B contracts, we draw the bid days according to $\widehat{F}_{B|Z}(b|z)$, with z being the characteristics of this contract observation. Then, we obtain the simulated cost types.[20] Using the simulated types can yield simulated bid costs for A+B and lane rental contracts according to (13) and (42), respectively. Third, we draw uncertainties from the estimated conditional distribution of uncertainty lognormal $(\widehat{\mu}, \widehat{\sigma}^2(z))$ with $\widehat{\sigma}(z) = z'\widehat{\psi}_\sigma$ and use the simulated types and uncertainties to calculate the actual working days for each A+B and lane rental contract. As a result, we calculate the bid cost, construction cost, commuter cost and social cost for each contract. By repeating this process 1000 times, we obtain the average cost outcomes for that contract observation. Fourth, we conduct the second and third steps for each observation of an A+B contract and obtain the average cost across all observations.

Prior to the counterfactual results, we first examine the fitness of the estimated model by comparing a number of sample moments and their simulated counterparts. This comparison can be used to investigate the relevance of our multistep estimated model. Table 4 indicates that the simulated bid cost and bid days at the bidder and contract levels are all close to what we observe in the data. Overall, the fit is good, although it slightly over-predicts the fraction of contracts that are finished late and under-predicts the fraction

[20] An asymptotically equivalent method is to directly draw cost types from the estimated conditional distribution of cost type $\widehat{F}_{\Theta|Z}(\theta|z)$. Given the finite sample size of the data, however, this alternative method would incur more finite-sample errors because the conditional distribution of cost type is estimated from the *estimated* cost types, while the conditional distribution of bid days is estimated from the *observed* bid days.

that are finished early.

[Insert Table 4 here]

Table 4: Sample and Simulated Moments of the Structural Model

	Data Mean	Simulated Mean
Bid Cost ($M)	20.50	21.76
Bid Days	199.17	200.45
Winning Bid Cost ($M)	18.65	19.02
Contract Days	230.96	233.79
Working Days	266.81	268.89
Fractions of Contracts Late	0.46	0.63
Fractions of Contracts Early	0.24	0.13

Note: Comparisons of observed and simulated moments for the structural model. The contract days is the number of days bid by the winning contractor. The second column contains moments observed in the data; the third column contains moments simulated 1000 times using the model estimates.

Table 5 reports the simulated and counterfactual costs for A+B and lane rental contracts, respectively. First, the average social cost for A+B contracts is $96.01 million, which is larger than the average social cost of $54.62 million for lane rental contracts. Hence, lane rental contracts can reduce the social cost by $41.39 million (43.11 percent) on average. This is consistent with the theoretical results that lane rental contracts are ex-post efficient while A+B contracts are not.

[Insert Table 5 here]

Table 5: Comparisons of Various Costs between A+B and Lane Rental Contracts

	Social Cost	Commuter Cost	Construction Cost	Bid Cost
A+B ($M)	96.01	78.60	17.41	19.02
Lane Rental ($M)	54.62	16.54	38.08	56.73
Absolute Change ($M)	41.39	62.06	20.67	37.71
Percentage Change	43.11	78.96	54.28	66.47

Note: The results are averaged across 1000 simulations and 205 A+B contracts. *Construction Cost* equals realized uncertainty ε times deterministic cost $c(x^A, \theta)$. *Commuter Cost* equals to daily cost c_s times actual working days x^A. *Social Cost* is the sum of construction cost and commuter cost.

Second, we analyze the two components of the social cost defined in (14). The average commuter cost for lane rental contracts is $16.54 million, which is only 21.04 percent of the average commuter cost of $78.60 million for A+B contracts. This result is also consistent with the theoretical prediction that the number of working days (and hence the commuter cost) under lane rental contracts is smaller than that under A+B contracts, as indicated by Figure 5. Consequently, the average construction cost under a lane rental contract is larger than that under an A+B contracts; the average construction cost for A+B contracts is $17.41 million, which is $20.67 million (54.28 percent) smaller than the average construction cost $38.08 million for a lane rental contract. In addition, the advantage gained from commuter cost in lane rental contracts outweighs its disadvantage in construction cost in terms of magnitude, so that the social cost of lane rental contracts is smaller than that of A+B contracts.

[Insert Figure 5 here]

Third, the average bid cost (procurement cost) for lane rental contracts is $56.73 million, which is larger than that for A+B contracts ($19.02 million). This can be explained as follows. On the one hand, lane rental contracts lead contractors to completely internalize the commuter cost through their bidding strategy. As shown in the equilibrium bid

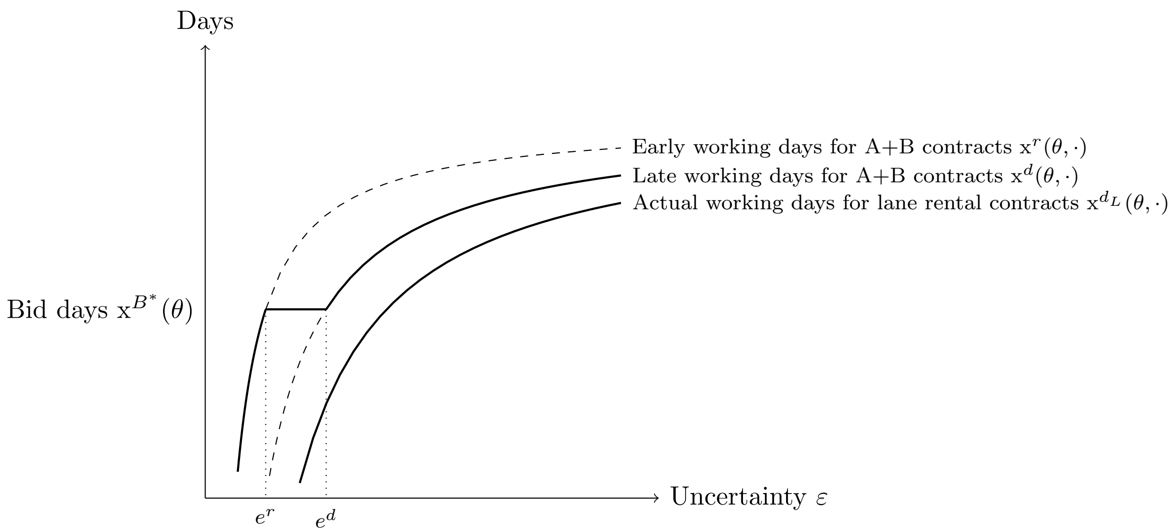

Figure 5: Comparison of actual working days between A+B and lane rental contracts
Note: Figure 5 plots the equilibrium relationship between actual working days and uncertainty in A+B and lane rental contracts, respectively, with $r < d < c_s = d_L$. The upper solid curve depicts the actual working days in A+B contracts; the lower solid curve depicts the actual working days in lane rental contracts.

(42), the term in the bracket is the social cost. Therefore, a larger social cost in a lane rental contract corresponds to a higher bid cost. On the other hand, a large project size can lead to a high bid cost for A+B and lane rental contracts. Since the average construction cost for lane rental contracts is larger than that for A+B contracts in Table 5, the average bid cost for lane rental contracts can be higher than that for A+B contracts.

Furthermore, Figure 5 suggests an important policy implication. If the incentives and disincentives were both equal to the daily commuter cost, i.e., $r = d = c_s$, the A+B contract design would also be ex-post efficient because its actual working days would be the same as in the ex-post efficient lane rental contract.[21] However, setting these equal would substantially increase construction costs, which would be passed on to Caltrans. Facing its budget constraint (e.g., Lewis and Bajari, 2011), in practice, the DoT might

[21] Although A+B contracts would be ex-post efficient under $r = d = c_s$, they differ from lane rental contracts in practice, since the latter require the bidder to quote a cost bid only and have no required completion date.

strike an appropriate balance between reducing commuter costs and alleviating budget pressures by setting $r < d$ in A+B contracts.[22]

6.4. Robustness

We investigate the robustness of the main results to an alternative parametric specification of the cost function.

Assumption 3.

(a) The cost function is $c(x, \theta) = \theta x^{\widetilde{\alpha}}$ *with* $\widetilde{\alpha} < 0$.

(b) The lower bound of type support is $\underline{\theta} = 1$.

The cost function specified in Part (a) of Assumption 3 is also used in the empirical literature such as Lewis and Bajari (2011) and Luo et al. (2018b). The cost parameter $\widetilde{\alpha} < 0$ is implied by Assumption 1. Part (b) is the same as in Assumption 2. Using arguments similar to those in Proposition 3 and Corollary 1, we obtain the identification of the model primitives as below.

Corollary 3. *Suppose that Assumptions 1 and 3 hold. Then, the cost parameter* $\widetilde{\alpha}$ *is identified, and the type distribution* $F_\Theta(\cdot)$ *and the uncertainty distribution* $F(\cdot)$ *are identified on the supports* $\mathcal{S}_\Theta$ *and* $\widetilde{\mathcal{S}}_\varepsilon$*, respectively. Moreover, if* $F(\cdot)$ *is parameterized, the set of its parameters is identified.*

Proof: See the Appendix.

The estimation and counterfactual procedures under the alternative cost function are similar to those under our main cost function except that we estimate a different cost

[22] The budget constraint is often an important concern for highway construction procurement in other states, such as Michigan, Minnesota, and Oklahoma (see, e.g., De Silva et al., 2009; Marion, 2009; Lewis and Bajari, 2014; Groeger, 2014).

function. Similar to (34), we estimate the cost parameter $\widetilde{\alpha}$ in terms of a heterogeneous quantile relationship implied by (A.71) in the Appendix

$$\widehat{\widetilde{\alpha}} = \underset{\widetilde{\alpha}}{\operatorname{argmin}} \frac{1}{J} \sum_{j=1}^{J} \left\{ \frac{1}{n_j} \sum_{i=1}^{n_j} \left[\widehat{v}_{ji} - x_{ji}^B M(\widetilde{\alpha}) \right]^2 \right\}, \tag{43}$$

where

$$\begin{aligned} M(\widetilde{\alpha}) &= c_{u_j} + \widehat{\kappa}_0(z_j)/\widetilde{\alpha} - r_j \widehat{F}(\widehat{e}^r(z_j)|z_j) - d_j(1 - \widehat{F}(\widehat{e}^d(z_j)|z_j)) \\ &+ \frac{\widetilde{\alpha}-1}{\widetilde{\alpha}} \left[r_j \widehat{F}(\widehat{e}^r(z_j)|z_j) \frac{\widehat{\mu}_r(z_j)}{\widehat{\mu}_B(z_j)} + d_j(1 - \widehat{F}(\widehat{e}^d(z_j)|z_j)) \frac{\widehat{\mu}_d(z_j)}{\widehat{\mu}_B(z_j)} \right]. \end{aligned}$$

Combining the above estimates, we obtain the type estimate for bidder i in contract j as $\widehat{\theta}_{ji} = \widehat{\kappa}_1(z_j)\widehat{\widetilde{\alpha}}^{-1} \left(X_{ji}^B\right)^{1-\widehat{\widetilde{\alpha}}}$, where $\widehat{\kappa}_1(z_j)$ is defined in the Appendix. Using the estimates of $\{(\widehat{\theta}_{ji})_{i=1}^{n_j}\}_{j=1}^{J}$, we can obtain the estimator of the conditional type distribution $F_{\Theta|Z}(\cdot|z)$ given $Z = z$.

The main results under the alternative cost function are very close to those under the main cost function. First, the estimate of the cost parameter $\widetilde{\alpha}$ is -0.214, which is significant with a standard error of 0.028.[23] The negative estimate of the cost parameter conforms to Assumption 3. The estimated marginal effect of working days on the cost, $\theta\widetilde{\alpha}x^{\widetilde{\alpha}-1}$, is also of magnitude 10^4 on average as in Section 5.2, given that the average estimate of the bidder's type is also 10^8 in magnitude.[24] The counterfactual results in Table 6 are qualitatively identical to those in Table 5. In particular, both the social cost and the commuter cost for lane rental contracts are much lower than those for A+B contracts.

[23] The estimates of parameters of the uncertainty distribution and cutoff uncertainty regression are identical to their counterparts in Table 3 because they are obtained using the same estimation procedure.

[24] The shape and magnitude of the estimated type distribution under the alternative cost function are very close to those in Figure 4. In addition, the fitness of the estimated model is also good. These results are available upon request.

[Insert Table 6 here]

Table 6: Comparisons of Various Costs between A+B and Lane Rental Contracts (Robustness)

	Social Cost	Commuter Cost	Construction Cost	Bid Cost
A+B ($M)	100.88	81.42	19.46	20.99
Lane Rental ($M)	56.40	17.65	38.75	60.51
Absolute Change ($M)	44.48	63.77	19.29	39.52
Percentage Change	44.09	78.32	49.78	65.31

Note: The results are averaged across 1000 simulations and 205 A+B contracts. *Construction Cost* equals realized uncertainty ε times deterministic cost $c(x^A, \theta)$. *Commuter Cost* equals to daily cost c_s times actual working days x^A. *Social Cost* is the sum of construction cost and commuter cost.

7. Concluding Remarks

This paper studies how uncertainty affects the efficiency of A+B procurement contracting with time incentives. We introduce a model with ex-post uncertainty to rationalize the deviation of actual working days from ex-ante bid days in the highway construction industry. The A+B contract design with uncertainty is not ex-post efficient. We show that the model components are identified from the bid days, actual working days and bid score. We use highway procurement contracts in California to estimate the model. Compared to A+B contracts, the counterfactual lane rental contract can substantially reduce the social cost and commuter cost in the presence of construction uncertainty. These results are stable across different specifications of the cost function. One future research direction is to extend the model by considering risk-averse bidders. Our econometric method might also be extended to other contract mechanisms with ex-post uncertainty.

Appendix

A. Proofs

Proof of Lemma 1.

We use the backward induction to analyze the equilibrium. First, we show the contractor's optimal actual working days in the second construction stage, given his bid pair (p^B, x^B) in the auction stage, contractor's type θ, and realization of uncertainty ε. In the second stage of construction, the winning contractor chooses the optimal actual working days to maximize the following payoff

$$\pi^{II}\left(x^A \,|\, p^B, x^B, \theta, \varepsilon\right) = p^B - TC = p^B - \varepsilon \cdot c(x^A, \theta) - K(x^A, x^B, r, d), \tag{A.1}$$

where the total cost

$$TC = \varepsilon \cdot c(x^A, \theta) + K(x^A, x^B, r, d),$$

and the incentive cost

$$K(x^A, x^B, r, d) = \mathbb{1}(x^A < x^B) \cdot r \cdot (x^A - x^B) + \mathbb{1}(x^A > x^B) \cdot d \cdot (x^A - x^B).$$

Since p^B is additively separable in (A.1), the optimal actual working days, which is a function of $(x^B, \theta, \varepsilon)$, is given by

$$\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = \underset{x^A}{\operatorname{argmin}} \{\varepsilon \cdot c(x^A, \theta) + K(x^A, x^B, r, d)\}. \tag{A.2}$$

Due to the discontinuity in the incentive cost $K(\cdot, x^B, r, d)$ at x^B, we need to define two cutoff levels of uncertainty

$$\varepsilon^r(\theta, x^B) = \frac{-r}{c_1(x^B, \theta)} \text{ and } \varepsilon^d(\theta, x^B) = \frac{-d}{c_1(x^B, \theta)}. \tag{A.3}$$

Since $r < d$ and $c_1(\cdot, \cdot) < 0$ in Assumption 1, we have

$$0 < \varepsilon^r(\theta, x^B) < \varepsilon^d(\theta, x^B). \tag{A.4}$$

Next, we show the main results: (i) if $\varepsilon \leq \varepsilon^r(\theta, x^B)$, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^r(\theta, \varepsilon)$ with

$$-\varepsilon \cdot c_1(x^r(\theta, \varepsilon), \theta) = r, \tag{A.5}$$

(ii) if $\varepsilon \geq \varepsilon^d(\theta, x^B)$, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^d(\theta, \varepsilon)$ such that

$$-\varepsilon \cdot c_1(x^d(\theta, \varepsilon), \theta) = d, \tag{A.6}$$

(iii) if $\varepsilon \in [\varepsilon^r(\theta, x^B), \varepsilon^d(\theta, x^B)]$, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^B$.

Let us consider case (i): $\varepsilon \leq \varepsilon^r(\theta, x^B)$. If the contractor completes the construction on time, i.e., $x^A = x^B$, the total cost is

$$TC^0 = \varepsilon \cdot c(x^B, \theta).$$

If the project completion is delayed, i.e., $x^A > x^B$, the total cost is

$$TC^+ = \varepsilon \cdot c(x^A, \theta) + d \cdot (x^A - x^B).$$

We will prove that when $\varepsilon \leq \varepsilon^r(\theta, x^B)$, $TC^0 < TC^+$ for any $x^A > x^B$, which implies that late completion is worse than on-time completion. Specifically,

$$\begin{aligned}
TC^0 - TC^+ &= \varepsilon \cdot c(x^B, \theta) - \left[\varepsilon \cdot c(x^A, \theta) + d \cdot (x^A - x^B)\right] \\
&= \varepsilon \left[c(x^B, \theta) - c(x^A, \theta)\right] - \varepsilon^d(\theta, x^B) \cdot c_1(x^B, \theta) \cdot (x^B - x^A) \\
&= \varepsilon \cdot c_1(y, \theta) \cdot (x^B - x^A) - \varepsilon^d(\theta, x^B) \cdot c_1(x^B, \theta) \cdot (x^B - x^A) \\
&= \left[\varepsilon \cdot c_1(y, \theta) - \varepsilon \cdot c_1(x^B, \theta) + \varepsilon \cdot c_1(x^B, \theta) - \varepsilon^d(\theta, x^B) \cdot c_1(x^B, \theta)\right] \cdot (x^B - x^A) \\
&= \Big(\underbrace{\varepsilon \left[c_1(y, \theta) - c_1(x^B, \theta)\right]}_{+} + \underbrace{(\varepsilon - \varepsilon^d(\theta, x^B))c_1(x^B, \theta)}_{+} \Big) \cdot \underbrace{(x^B - x^A)}_{-} \\
&< 0, \qquad \text{(A.7)}
\end{aligned}$$

where the second equality is from (A.3), the third equality follows the mean-value theorem with $x^B < y < x^A$, the first part in the fifth equality is positive due to the convexity of cost function in Assumption 1, the second part is positive due to $\varepsilon < \varepsilon^r(\theta, x^B) < \varepsilon^d(\theta, x^B)$ and assumption $c_1(\cdot, \cdot) < 0$, and the last part is negative due to $x^A > x^B$.

Accordingly, we just need to show that early completion at the actual working days $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^r(\theta, \varepsilon)$ is better than on-time completion, that is, $TC^0 > TC^r$, where

$$TC^r = \varepsilon \cdot c(x^r(\theta, \varepsilon), \theta) + r \cdot (x^r(\theta, \varepsilon) - x^B).$$

Specifically,

$$
\begin{aligned}
TC^r - TC^0 &= \varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) + r \cdot (x^r(\theta,\varepsilon) - x^B) - \varepsilon \cdot c(x^B,\theta) \\
&= \varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) - \varepsilon \cdot c_1(x^r(\theta,\varepsilon),\theta) \cdot (x^r(\theta,\varepsilon) - x^B) - \varepsilon \cdot c(x^B,\theta) \\
&= \varepsilon \Big[c(x^r(\theta,\varepsilon),\theta) - c(x^B,\theta) - c_1(x^r(\theta,\varepsilon),\theta) \cdot (x^r(\theta,\varepsilon) - x^B) \Big] \\
&= \varepsilon \Big[c_1(y,\theta) \cdot (x^r(\theta,\varepsilon) - x^B) - c_1(x^r(\theta,\varepsilon),\theta) \cdot (x^r(\theta,\varepsilon) - x^B) \Big] \\
&= \underbrace{\varepsilon}_{+} \underbrace{\Big[c_1(y,\theta) - c_1(x^r(\theta,\varepsilon),\theta) \Big]}_{+} \cdot \underbrace{(x^r(\theta,\varepsilon) - x^B)}_{-} \\
&< 0,
\end{aligned}
$$

where the second equality is from (A.5), the fourth equality follows the mean-value theorem with $x^r(\theta,\varepsilon) < y < x^B$, the second part in the fifth equality is positive due to the convexity of cost function, and the last part

$$x^r(\theta,\varepsilon) < x^B \tag{A.8}$$

is implied by the definition of $\widetilde{\varepsilon}^r(\theta, x^B)$ in (A.3), convexity of cost function, and $\varepsilon < \varepsilon^r(\theta, x^B)$.

Using similar arguments for case (i), we can show $\widetilde{x}^{A^*} = x^d(\theta,\varepsilon)$ for case (ii): $\varepsilon > \varepsilon^d$. If the contractor chooses early completion, i.e, $x^A < x^B$, the total cost is

$$TC^- = \varepsilon \cdot c(x^A,\theta) + r \cdot (x^A - x^B).$$

Similarly, we can prove that when $\varepsilon > \widetilde{\varepsilon}^d(\theta, x^B)$, $TC^0 < TC^-$ for any $x^A < x^B$, which implies that early completion is worse than on-time completion. Specifically,

$$
\begin{aligned}
TC^0 - TC^- &= \varepsilon \cdot c(x^B, \theta) - \left[\varepsilon \cdot c(x^A, \theta) + r \cdot (x^A - x^B)\right] \\
&= \varepsilon \left[c(x^B, \theta) - c(x^A, \theta)\right] - \varepsilon^r(\theta, x^B) \cdot c_1(x^B, \theta) \cdot (x^B - x^A) \\
&= \varepsilon \cdot c_1(y, \theta) \cdot (x^B - x^A) - \varepsilon^r(\theta, x^B) \cdot c_1(x^B, \theta) \cdot (x^B - x^A) \\
&= \left[\varepsilon \cdot c_1(y, \theta) - \varepsilon \cdot c_1(x^B, \theta) + \varepsilon \cdot c_1(x^B, \theta) - \varepsilon^r(\theta, x^B) \cdot c_1(x^B, \theta)\right] \cdot (x^B - x^A) \\
&= \Big(\underbrace{\varepsilon \left[c_1(y, \theta) - c_1(x^B, \theta)\right]}_{-} + \underbrace{(\varepsilon - \varepsilon^r(\theta, x^B))c_1(x^B, \theta)}_{-}\Big) \cdot \underbrace{(x^B - x^A)}_{+} \\
&< 0, \qquad \text{(A.9)}
\end{aligned}
$$

where $x^A < y < x^B$. Accordingly, we just need to show that late completion at the actual working days $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^d(\theta, \varepsilon)$ is better than on-time completion, that is, $TC^0 > TC^d$, where

$$
TC^d = \varepsilon \cdot c(x^d(\theta, \varepsilon), \theta) + d \cdot (x^d(\theta, \varepsilon) - x^B).
$$

Using similar arguments, we obtain

$$
\begin{aligned}
TC^d - TC^0 &= \varepsilon \cdot c(x^d(\theta, \varepsilon), \theta) + r \cdot (x^d(\theta, \varepsilon) - x^B) - \varepsilon \cdot c(x^B, \theta) \\
&= \varepsilon \cdot c(x^d(\theta, \varepsilon), \theta) - \varepsilon \cdot c_1(x^d(\theta, \varepsilon), \theta) \cdot (x^d(\theta, \varepsilon) - x^B) - \varepsilon \cdot c(x^B, \theta) \\
&= \varepsilon \Big[c(x^d(\theta, \varepsilon), \theta) - c(x^B, \theta) - c_1(x^d(\theta, \varepsilon), \theta) \cdot (x^d(\theta, \varepsilon) - x^B)\Big] \\
&= \varepsilon \Big[c_1(y, \theta) \cdot (x^d(\theta, \varepsilon) - x^B) - c_1(x^d(\theta, \varepsilon), \theta) \cdot (x^d(\theta, \varepsilon) - x^B)\Big] \\
&= \underbrace{\varepsilon}_{+} \underbrace{\Big[c_1(y, \theta) - c_1(x^d(\theta, \varepsilon), \theta)\Big]}_{-} \cdot \underbrace{(x^d(\theta, \varepsilon) - x^B)}_{+} \\
&< 0,
\end{aligned}
$$

where $x^B < y < x^d$, and the last part

$$x^d(\theta, \varepsilon) > x^B \tag{A.10}$$

is implied by the definition of $\varepsilon^d(\theta, x^B)$ in (A.3), convexity of cost function, and $\varepsilon > \varepsilon^d(\theta, x^B)$.

Next, we prove $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^B$ for case (iii): $\varepsilon^r(\theta, x^B) \leq \varepsilon \leq \varepsilon^d(\theta, x^B)$. On the one hand, for any $x^A > x^B$, as (A.7) shows, $TC^0 - TC^+ < 0$. On the other hand, for any $x^A < x^B$, as (A.9) shows, $TC^0 - TC^- < 0$. Therefore, on-time completion is optimal for the contractor, i.e, $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon) = x^B$.

Last, combing (A.8) with (A.10) leads to $x^r(\theta, \varepsilon) \leq x^B \leq x^d(\theta, \varepsilon)$. The proof is complete.

Proof of Proposition 1.

Using the results of optimal actual working days in Lemma 1, we derive the equilibrium of our model by analyzing the first-stage optimal decision on the bid pair of cost and working days. In the first stage, the bidder with type θ quotes the optimal pair of cost and working days to maximize his expected payoff

$$\begin{aligned} \pi(p^B, x^B \mid \theta) \quad &= \quad \left\{ p^B - \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), \theta) + K(\widetilde{x}^{A^*}(x^B, \theta, \varepsilon), x^B, r, d) \Big] \right\} \\ &\times \quad \Pr\Big(\text{win} \;\Big|\; s = p^B + c_u x^B \Big), \end{aligned}$$

that is,

$$\left(p^{B^*}(\theta), x^{B^*}(\theta)\right) = \underset{p^B, x^B}{\operatorname{argmax}} \; \pi\left(p^B, x^B \,|\, \theta\right),$$

where $\mathbb{E}_\varepsilon$ is the expectation with respect to ε, $\Pr(\text{win}|s = p^B + c_u x^B)$ is the probability of winning the contract given his bid score $s = p^B + c_u x^B$, and $\widetilde{x}^{A^*}(x^B, \theta, \varepsilon)$ is defined in Lemma 1.

First, we follow Che (1993) to show that for any type θ, the equilibrium bid days $x^{B^*}(\theta)$ can be determined separately from the choice of score through

$$\text{(A.11)}\quad x^{B^*}(\theta) = arg\min_{x^B}\left\{c_u x^B + \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^B,\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(x^B,\theta,\varepsilon),x^B,r,d)\Big]\right\}.$$

Suppose the contractor with type θ bids $(\widetilde{p}^B,\widetilde{x}^B)$ where $\widetilde{x}^B \neq x^{B^*}(\theta)$. We only need to show that the contractor is better-off if he chooses bid days $x^{B^*}(\theta)$ and bid cost $p^{B^*}(\theta)$ with

$$p^{B^*}(\theta) = \widetilde{p}^B + c_u(\widetilde{x}^B - x^{B^*}(\theta)).$$

Note that the scores are identical in both choices due to

$$s^*(\theta) \equiv p^{B^*}(\theta) + c_u x^{B^*}(\theta) = \widetilde{p}^B + c_u(\widetilde{x}^B - x^{B^*}(\theta)) + c_u x^{B^*}(\theta) = \widetilde{p}^B + c_u\widetilde{x}^B.$$

The difference of contractor's expected payoff between $(\widetilde{p}^B,\widetilde{x}^B)$ and $(p^{B^*}(\theta), x^{B^*}(\theta))$ is

$$\begin{aligned}
&\pi(p^{B^*}(\theta), x^{B^*}(\theta) \mid \theta) - \pi(\widetilde{p}^B,\widetilde{x}^B \mid \theta)\\
=&\Big\{\widetilde{p}^B + c_u(\widetilde{x}^B - x^{B^*}(\theta)) - \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^{B^*}(\theta),\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(x^{B^*}(\theta),\theta,\varepsilon),x^B,r,d)\big]\\
&- \widetilde{p}^B + \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(\widetilde{x}^B,\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(\widetilde{x}^B,\theta,\varepsilon),x^B,r,d)\big]\Big\} \times \Pr(\text{win} \mid s^*(\theta))\\
=&\Big\{\Big(c_u\widetilde{x}^B + \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(\widetilde{x}^B,\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(\widetilde{x}^B,\theta,\varepsilon),x^B,r,d)\big]\Big)\\
&- \Big(c_u x^{B^*}(\theta) + \mathbb{E}_\varepsilon\big[\varepsilon \cdot c(\widetilde{x}^{A^*}(x^{B^*}(\theta),\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(x^{B^*}(\theta),\theta,\varepsilon),x^B,r,d)\big]\Big)\Big\} \times \Pr(\text{win} \mid s^*(\theta))\\
>&0,
\end{aligned}$$

where the last inequality comes from the determination of $x^{B^*}(\theta)$ in (A.11) and that the winning probability is positive as shown in Che (1993).

Second, we prove $dx^{B^*}(\theta)/d\theta > 0$. The first-order condition with respect to x^B in the objective function (A.11) implies

$$
\begin{aligned}
0 =& c_u + \frac{\partial}{\partial x^B}\left[\int_{\varepsilon \le \varepsilon^r(\theta, x^B)} \left[\varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) + r \cdot (x^r(\theta,\varepsilon) - x^B)\right] dF(\varepsilon)\right] \\
& + \frac{\partial}{\partial x^B}\left[\int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot c(x^B,\theta) dF(\varepsilon)\right] \\
& + \frac{\partial}{\partial x^B}\left[\int_{\varepsilon \ge \varepsilon^d(\theta, x^B)} \left[\varepsilon \cdot c(x^d(\theta,\varepsilon),\theta) + d \cdot (x^d(\theta,\varepsilon) - x^B)\right] dF(\varepsilon)\right] \\
=& c_u + \left\{\varepsilon^r(\theta,x^B) c(x^r(\theta,\varepsilon^r(\theta,x^B)),\theta) + r\left[x^r(\theta,\varepsilon^r(\theta,x^B)) - x^B\right]\right\} f(\varepsilon^r(\theta,x^B)) \frac{\partial \varepsilon^r(\theta,x^B)}{\partial x^B} \\
& + \int_{\varepsilon \le \varepsilon^r(\theta,x^B)} \frac{\partial}{\partial x^B}\left[\varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) + r \cdot (x^r(\theta,\varepsilon) - x^B)\right] f(\varepsilon) d\varepsilon \\
& + \left[\varepsilon^d(\theta,x^B) c(x^B,\theta) f(\varepsilon^d(\theta,x^B))\right] \partial\varepsilon^d(\theta,x^B)/\partial x^B \\
& - \left[\varepsilon^r(\theta,x^B) c(x^B,\theta) f(\varepsilon^r(\theta,x^B))\right] \partial\varepsilon^r(\theta,x^B)/\partial x^B + \int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot \frac{\partial c(x^B,\theta)}{\partial x^B} f(\varepsilon) d\varepsilon \\
& - \left\{\varepsilon^d(\theta,x^B) c(x^d(\theta,\varepsilon^d(\theta,x^B)),\theta) + d\left[x^d(\theta,\varepsilon^d(\theta,x^B)) - x^B\right]\right\} f(\varepsilon^d(\theta,x^B)) \frac{\partial \varepsilon^d(\theta,x^B)}{\partial x^B} \\
& + \int_{\varepsilon \ge \varepsilon^d(\theta,x^B)} \frac{\partial}{\partial x^B}\left[\varepsilon \cdot c(x^d(\theta,\varepsilon),\theta) + d \cdot (x^d(\theta,\varepsilon) - x^B)\right] f(\varepsilon) d\varepsilon \\
=& c_u + \int_{\varepsilon \le \varepsilon^r(\theta,x^B)} \frac{\partial}{\partial x^B}\left[\varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) + r \cdot (x^r(\theta,\varepsilon) - x^B)\right] f(\varepsilon) d\varepsilon \\
& + \int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot \frac{\partial c(x^B,\theta)}{\partial x^B} f(\varepsilon) d\varepsilon \\
& + \int_{\varepsilon \ge \varepsilon^d(\theta,x^B)} \frac{\partial}{\partial x^B}\left[\varepsilon \cdot c(x^d(\theta,\varepsilon),\theta) + d \cdot (x^d(\theta,\varepsilon) - x^B)\right] f(\varepsilon) d\varepsilon \\
=& c_u - \int_{\varepsilon \le \varepsilon^r(\theta,x^B)} r f(\varepsilon) d\varepsilon + \int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot c_1(x^B,\theta) f(\varepsilon) d\varepsilon - \int_{\varepsilon \ge \varepsilon^d(\theta,x^B)} d f(\varepsilon) d\varepsilon
\end{aligned}
$$

(A.12)

$$
= c_u + \int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon \cdot c_1(x^B,\theta) f(\varepsilon) d\varepsilon - rF(\varepsilon^r(\theta,x^B)) - d\left[1 - F(\varepsilon^d(\theta,x^B))\right],
$$

where the third equality follows $x^r(\theta, \varepsilon^r(\theta, x^B)) = x^B$ and $x^d(\theta, \varepsilon^d(\theta, x^B)) = x^B$. Hence, the equilibrium bid days $x^{B^*}(\theta)$ satisfies

(A.13)
$$-c_u = \int_{\varepsilon^r(\theta, x^{B^*}(\theta))}^{\varepsilon^d(\theta, x^{B^*}(\theta))} \varepsilon \cdot c_1(x^{B^*}(\theta), \theta) f(\varepsilon) d\varepsilon - rF(\varepsilon^r(\theta, x^{B^*}(\theta))) - d\big[1 - F(\varepsilon^d(\theta, x^{B^*}(\theta)))\big].$$

Denote $x^{B^*} = x^{B^*}(\theta)$ for simplicity of exposition whenever there is no ambiguity. Taking the first derivative of both sides of (A.13) with respect to θ implies

$$\begin{aligned}
0 \;=\; & \varepsilon^d(\theta, x^{B^*}) c_1(x^{B^*}, \theta) f(\varepsilon^d(\theta, x^{B^*})) \left(\frac{\partial \varepsilon^d(\theta, x^{B^*})}{\partial \theta} + \frac{\partial \varepsilon^d(\theta, x^{B^*})}{\partial x^{B^*}} \frac{dx^{B^*}}{d\theta} \right) \\
- \; & \varepsilon^r(\theta, x^{B^*}) c_1(x^{B^*}, \theta) f(\varepsilon^r(\theta, x^{B^*})) \left(\frac{\partial \varepsilon^r(\theta, x^{B^*})}{\partial \theta} + \frac{\partial \varepsilon^r(\theta, x^{B^*})}{\partial x^{B^*}} \frac{dx^{B^*}}{d\theta} \right) \\
+ \; & \int_{\varepsilon^r(\theta, x^{B^*})}^{\varepsilon^d(\theta, x^{B^*})} \varepsilon \cdot \left[c_{11}(x^{B^*}, \theta) \frac{dx^{B^*}}{d\theta} + c_{12}(x^{B^*}, \theta) \right] f(\varepsilon) d\varepsilon \\
- \; & r f(\varepsilon^r(\theta, x^{B^*})) \left(\frac{\partial \varepsilon^r(\theta, x^{B^*})}{\partial \theta} + \frac{\partial \varepsilon^r(\theta, x^{B^*})}{\partial x^{B^*}} \frac{dx^{B^*}}{d\theta} \right) \\
+ \; & d f(\varepsilon^d(\theta, x^{B^*})) \left(\frac{\partial \varepsilon^d(\theta, x^{B^*})}{\partial \theta} + \frac{\partial \varepsilon^d(\theta, x^{B^*})}{\partial x^{B^*}} \frac{dx^{B^*}}{d\theta} \right) \\
= \; & \int_{\varepsilon^r(\theta, x^{B^*})}^{\varepsilon^d(\theta, x^{B^*})} \varepsilon \cdot \left[c_{11}(x^{B^*}, \theta) \frac{dx^{B^*}}{d\theta} + c_{12}(x^{B^*}, \theta) \right] f(\varepsilon) d\varepsilon, \qquad \text{(A.14)}
\end{aligned}$$

where the second equality follows the definitions of $\varepsilon^r(\theta, x^{B^*}(\theta))$ and $\varepsilon^d(\theta, x^{B^*}(\theta))$. Since $\varepsilon > 0$, it follows

$$c_{11}(x^{B^*}(\theta), \theta) \frac{dx^{B^*}(\theta)}{d\theta} + c_{12}(x^{B^*}(\theta), \theta) = 0. \qquad \text{(A.15)}$$

Due to $c_{11}(\cdot, \cdot) > 0$ and $c_{12}(\cdot, \cdot) < 0$, we obtain

$$\frac{dx^{B^*}(\theta)}{d\theta} = -\frac{c_{12}(x^{B^*}(\theta), \theta)}{c_{11}(x^{B^*}(\theta), \theta)} > 0. \qquad \text{(A.16)}$$

In addition, we prove that for any type θ, in equilibrium the two cutoff levels of uncertainty are constant. Based on the equilibrium bid days $x^{B^*}(\theta)$, the two cutoff levels in equilibrium depend only on type θ in the way

$$\varepsilon^r(\theta) \equiv \varepsilon^r(\theta, x^{B^*}(\theta)) \text{ and } \varepsilon^d(\theta) \equiv \varepsilon^d(\theta, x^{B^*}(\theta)).$$

According to definitions of $\varepsilon^r(\theta)$ and $\varepsilon^d(\theta)$, in equilibrium it satisfies

$$\varepsilon^r(\theta)c_1(x^{B^*}(\theta), \theta) = -r \text{ and } \varepsilon^d(\theta)c_1(x^{B^*}(\theta), \theta) = -d.$$

Taking the first derivatives with respect to θ on both sides leads to

$$\text{(A.17)} \qquad \frac{d\varepsilon^r(\theta)}{d\theta}c_1(x^{B^*}(\theta), \theta) + \varepsilon^r(\theta)\left[c_{11}(x^{B^*}(\theta), \theta)\frac{dx^{B^*}(\theta)}{d\theta} + c_{12}(x^{B^*}(\theta), \theta)\right] = 0$$

and

$$\text{(A.18)} \qquad \frac{d\varepsilon^d(\theta)}{d\theta}c_1(x^{B^*}(\theta), \theta) + \varepsilon^d(\theta)\left[c_{11}(x^{B^*}(\theta), \theta)\frac{dx^{B^*}(\theta)}{d\theta} + c_{12}(x^{B^*}(\theta), \theta)\right] = 0.$$

Combining (A.15) with $c_1(\cdot, \cdot) < 0$ implies

$$\frac{d\varepsilon^r(\theta)}{d\theta} = 0 \text{ and } \frac{d\varepsilon^d(\theta)}{d\theta} = 0,$$

that is,

$$\text{(A.19)} \qquad \varepsilon^r(\theta) \equiv e^r \text{ and } \varepsilon^d(\theta) \equiv e^d \text{ for any } \theta,$$

where both e^r and e^d are constant with $e^r < e^d$ implied by (A.4). As a result, using (3) in Lemma 1 we obtain the equilibrium actual working days

$$(A.20) \qquad x^{A^*}(\theta,\varepsilon) = \widetilde{x}^{A^*}(x^{B^*}(\theta),\theta,\varepsilon) = \begin{cases} x^r(\theta,\varepsilon) & \textit{if } \ \varepsilon \le e^r, \\ x^{B^*}(\theta) & \textit{if } \ \varepsilon \in [e^r, e^d], \\ x^d(\theta,\varepsilon) & \textit{if } \ \varepsilon \ge e^d. \end{cases}$$

Third, we derive the equilibrium bid of cost $p^{B^*}(\theta)$ by using the result on the unique symmetric monotone Bayesian Nash Equilibrium (psBNE) in the standard literature (see, Krishna, 2009). To do so, we start with the symmetric and increasing bidding strategy for optimal score $s_i^* = s(v_i)$ for the pseudo-type v_i of contractor $i \in \{1, \cdots, N\}$ such that

$$(A.21) \qquad s_i^* = \operatorname*{argmax}_b \Big\{ \big(b - v_i\big) \times \Pr(\text{win}_i \mid b) \Big\},$$

where $\Pr(\text{win}_i \mid b)$ denotes the winning probability of contractor i when his score is b, and

$$\Pr(\text{win}_i \mid b) = \Pi_{j \neq i} \Pr(b_j \ge b) = \Pi_{j \neq i} \Pr(v_j \ge s^{-1}(b)) = [1 - F_V(s^{-1}(b))]^{N-1},$$

where $F_V(\cdot)$ is the cumulative distribution function of pseudo-type V. With a slight abuse of notation, we define s_i^* as $s(v_i)$ rather than $s(v(\theta_i))$ to emphasize that here the optimal score is a function of pseudo-type. The first-order condition of (A.21) with respect to b yields

$$(A.22) \quad -(N-1)(s_i^* - v_i)[1 - F_V(s^{-1}(s_i^*))]^{N-2} \frac{f_V(s^{-1}(s_i^*))}{s'(s^{-1}(s_i^*))} + [1 - F_V(s^{-1}(s_i^*))]^{N-1} = 0.$$

Due to the symmetry of equilibrium, we drop the subscript i for simplicity. As a result,

$$\begin{aligned}\frac{d}{dv}\big\{[1-F_V(v)]^{N-1}s(v)\big\} &= -(N-1)s(v)[1-F_V(v)]^{N-2}f_V(v)+[1-F_V(v)]^{N-1}s'(v)\\ &= -(N-1)v[1-F_V(v)]^{N-2}f_V(v)\\ &= v\frac{d}{dv}\big\{[1-F_V(v)]^{N-1}\big\},\end{aligned} \tag{A.23}$$

where the second equality follows (A.22). Integrating by part on both sides of (A.23) with the boundary condition $s(\overline{v}) = \overline{v}$ yields

$$s(v) = v + \int_v^{\overline{v}} \left[\frac{1-F_V(t)}{1-F_V(v)}\right]^{N-1} dt > v, \tag{A.24}$$

where $\overline{v}$ is the upper bound of pseudo-type. Furthermore, the first-order derivative of (A.24) implies

$$s'(v) = (N-1)(s(v)-v)\frac{f_V(v)}{1-F_V(v)} > 0. \tag{A.25}$$

Next, we prove $v'(\theta) > 0$ and in turn obtain $F_V(v) = F_\Theta(\theta)$, which is critical for the equilibrium bid of cost $p^{B^*}(\theta)$. Recall the definition

$$v(\theta) = \min_{x^B}\left\{c_u x^B + \mathbb{E}_\varepsilon\Big[\varepsilon\cdot c(\widetilde{x}^{A^*}(x^B,\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(x^B,\theta,\varepsilon),x^B,r,d)\Big]\right\}. \tag{A.26}$$

Then, using the envelope theorem leads to

$$\begin{aligned}
v'(\theta) &= \partial\Big\{c_u x^{B^*} + \mathbb{E}_\varepsilon\big[\varepsilon\cdot c(\widetilde{x}^{A^*}(x^{B^*},\theta,\varepsilon),\theta) + K(\widetilde{x}^{A^*}(x^{B^*},\theta,\varepsilon),x^{B^*},r,d)\big]\Big\}/\partial\theta \\
&= \partial\Big\{\int_{\varepsilon\le\varepsilon^r(\theta,x^{B^*})}\big[\varepsilon\cdot c(x^r(\theta,\varepsilon),\theta) + r(x^r(\theta,\varepsilon)-x^{B^*})\big]f(\varepsilon)d\varepsilon\Big\}/\partial\theta \\
&+ \partial\Big\{\int_{\varepsilon^r(\theta,x^{B^*})}^{\varepsilon^d(\theta,x^{B^*})}\varepsilon\cdot c(x^{B^*},\theta)f(\varepsilon)d\varepsilon\Big\}/\partial\theta \\
&+ \partial\Big\{\int_{\varepsilon\ge\varepsilon^d(\theta,x^{B^*})}\big[\varepsilon\cdot c(x^d(\theta,\varepsilon),\theta) + d(x^d(\theta,\varepsilon)-x^{B^*})\big]f(\varepsilon)d\varepsilon\Big\}/\partial\theta \\
&= \varepsilon^r\cdot c(x^r(\theta,\varepsilon^r),\theta)f(\varepsilon^r)\partial\varepsilon^r(\theta,x^{B^*})/\partial\theta \\
&+ \int_{\varepsilon\le\varepsilon^r(\theta,x^{B^*})}\Big\{\varepsilon\big[c_1(x^r(\theta,\varepsilon),\theta)\partial x^r(\theta,\varepsilon)/\partial\theta + c_2(x^r(\theta,\varepsilon),\theta)\big] + r\partial x^r(\theta,\varepsilon)/\partial\theta\Big\}dF(\varepsilon) \\
&+ \varepsilon^d\cdot c(x^{B^*},\theta)f(\varepsilon^d)\partial\varepsilon^d(\theta,x^{B^*})/\partial\theta - \varepsilon^r\cdot c(x^{B^*},\theta)f(\varepsilon^r)\partial\varepsilon^r(\theta,x^{B^*})/\partial\theta \\
&+ \int_{\varepsilon^r(\theta,x^{B^*})}^{\varepsilon^d(\theta,x^{B^*})}\Big\{\varepsilon\cdot c_2(x^{B^*},\theta)\Big\}dF(\varepsilon) - \varepsilon^d\cdot c(x^d(\theta,\varepsilon^d),\theta)f(\varepsilon^d)\partial\varepsilon^d(\theta,x^{B^*})/\partial\theta \\
&+ \int_{\varepsilon\ge\varepsilon^d(\theta,x^{B^*})}\Big\{\varepsilon\big[c_1(x^d(\theta,\varepsilon),\theta)\partial x^d(\theta,\varepsilon)/\partial\theta + c_2(x^d(\theta,\varepsilon),\theta)\big] + d\partial x^d(\theta,\varepsilon)/\partial\theta\Big\}dF(\varepsilon) \\
&= \int_{\varepsilon\le\varepsilon^r(\theta,x^{B^*})}\Big\{\varepsilon\big[c_1(x^r(\theta,\varepsilon),\theta)\partial x^r(\theta,\varepsilon)/\partial\theta + c_2(x^r(\theta,\varepsilon),\theta)\big] + r\partial x^r(\theta,\varepsilon)/\partial\theta\Big\}dF(\varepsilon) \\
&+ \int_{\varepsilon^r(\theta,x^{B^*})}^{\varepsilon^d(\theta,x^{B^*})}\Big\{\varepsilon\cdot c_2(x^{B^*},\theta)\Big\}dF(\varepsilon) \\
&+ \int_{\varepsilon\ge\varepsilon^d(\theta,x^{B^*})}\Big\{\varepsilon\big[c_1(x^d(\theta,\varepsilon),\theta)\partial x^d(\theta,\varepsilon)/\partial\theta + c_2(x^d(\theta,\varepsilon),\theta)\big] + d\partial x^d(\theta,\varepsilon)/\partial\theta\Big\}dF(\varepsilon) \\
&= \int_{\varepsilon\le\varepsilon^r(\theta,x^{B^*})}\varepsilon\cdot c_2(x^r(\theta,\varepsilon),\theta)dF(\varepsilon) + \int_{\varepsilon^r(\theta,x^{B^*})}^{\varepsilon^d(\theta,x^{B^*})}\varepsilon\cdot c_2(x^{B^*},\theta)dF(\varepsilon) \\
&\int_{\varepsilon\ge\varepsilon^d(\theta,x^{B^*})}\varepsilon\cdot c_2(x^d(\theta,\varepsilon),\theta)dF(\varepsilon) > 0, \qquad \text{(A.27)}
\end{aligned}$$

where the third equality and fourth equality follow $x^r(\theta,\varepsilon^r) = x^{B^*}$ and $x^d(\theta,\varepsilon^r) = x^{B^*}$, and the fifth equality is obtained as follows. For expositional simplicity we denote $x^r = x^r(\theta,\varepsilon)$ and $x^d = x^d(\theta,\varepsilon)$ whenever there is no ambiguity. Combining Assumption 1 with

$-\varepsilon \cdot c_1(x^r(\theta,\varepsilon),\theta) = r$ implies

$$\frac{\partial x^r(\theta,\varepsilon)}{\partial\theta} = -\frac{c_{12}(x^r,\theta)}{c_{11}(x^r,\theta)} > 0 \text{ and } \frac{\partial x^r(\theta,\varepsilon)}{\partial\varepsilon} = -\frac{c_1(x^r,\theta)}{\varepsilon\cdot c_{11}(x^r,\theta)} > 0.$$

Similarly, it can be shown that $\partial x^d(\theta,\varepsilon)/\partial\theta > 0$ and $\partial x^d(\theta,\varepsilon)/\partial\varepsilon > 0$. The integrand in the first integral of the fourth equality equals

$$\varepsilon\left[-c_1(x^r,\theta)\frac{c_{12}(x^r,\theta)}{c_{11}(x^r,\theta)} + c_2(x^r,\theta)\right] + \varepsilon\cdot c_1(x^r,\theta)\frac{c_{12}(x^r,\theta)}{c_{11}(x^r,\theta)} = \varepsilon\cdot c_2(x^r,\theta) > 0,$$

where the last inequality follows $\varepsilon > 0$ and $c_2(\cdot,\cdot) > 0$ in Assumption 1. Similarly, the integrand in the third integral of the fourth equality equals $\varepsilon\cdot c_2(x^d,\theta) > 0$.

Using the property $F_V(v) = F_\Theta(\theta)$ implied by $v'(\theta) > 0$, the integral with respect to θ in (A.24) and $v'(\theta)$ in (A.27) yields the equilibrium bid of cost $p^{B^*}(\theta)$. Specifically, since

$$\begin{aligned} s(v(\theta)) &= c_u x^{B^*}(\theta) + \mathbb{E}_\varepsilon\Big[\varepsilon\cdot c(x^{A^*}(\theta,\varepsilon),\theta) + K(x^{A^*}(\theta,\varepsilon), x^{B^*}(\theta), r, d)\Big] \\ &+ \int_\theta^{\bar\theta} \mathbb{E}_\varepsilon[\varepsilon\cdot c_2(x^{A^*}(t,\varepsilon),t)]\left[\frac{1-F_\Theta(t)}{1-F_\Theta(\theta)}\right]^{N-1} dt, \end{aligned}$$

according to the scoring rule, we obtain

$$\begin{aligned} p^{B^*}(\theta) &= s(v(\theta)) - c_u x^{B^*}(\theta) = \mathbb{E}_\varepsilon\Big[\varepsilon\cdot c(x^{A^*}(\theta,\varepsilon),\theta) + K(x^{A^*}(\theta,\varepsilon), x^{B^*}(\theta), r, d)\Big] \\ &+ \int_\theta^{\bar\theta} \mathbb{E}_\varepsilon[\varepsilon\cdot c_2(x^{A^*}(t,\varepsilon),t)]\left[\frac{1-F_\Theta(t)}{1-F_\Theta(\theta)}\right]^{N-1} dt. \end{aligned} \tag{A.28}$$

Based on (A.25) and (A.27), we obtain $s'(\theta) = s'(v)v'(\theta) > 0$. Since $dx^{B^*}(\theta)/d\theta > 0$ and $dp^{B^*}(\theta)/d\theta = s'(\theta) - c_u dx^{B^*}(\theta)/d\theta$, it is not necessarily $dp^{B^*}(\theta)/d\theta > 0$. The proof is complete.

Proof of Proposition 2

In the text we have shown that the A+B contract with uncertainty is not ex-post efficient. Now we prove that the A+B contract with uncertainty may be ex-ante efficient. According to Proposition 1, the social welfare in equilibrium is given by

$$W^*(\theta,\varepsilon)=\begin{cases}V_c-\varepsilon\cdot c(x^r(\theta,\varepsilon),\theta)-c_s x^r(\theta,\varepsilon) & \textit{if}\ \ \varepsilon\le e^r,\\ V_c-\varepsilon\cdot c(x^{B^*}(\theta),\theta)-c_s x^{B^*}(\theta) & \textit{if}\ \ e^r\le\varepsilon\le e^d,\\ V_c-\varepsilon\cdot c(x^d(\theta,\varepsilon),\theta)-c_s x^d(\theta,\varepsilon) & \textit{if}\ \ \varepsilon\ge e^d.\end{cases}\tag{A.29}$$

Using (A.5) and (A.6) leads to

(A.30)

$$\frac{\partial W^*(\theta,\varepsilon)}{\partial\theta}=\begin{cases}(r-c_s)\frac{\partial x^r(\theta,\varepsilon)}{\partial\theta}-\varepsilon\cdot c_2(x^r(\theta,\varepsilon),\theta) & \textit{if}\ \ \varepsilon\le e^r,\\ -\left(\varepsilon\cdot c_1(x^{B^*}(\theta),\theta)+c_s\right)\frac{\partial x^{B^*}(\theta)}{\partial\theta}-\varepsilon\cdot c_2(x^{B^*}(\theta),\theta) & \textit{if}\ \ e^r\le\varepsilon\le e^d,\\ (d-c_s)\frac{\partial x^d(\theta,\varepsilon)}{\partial\theta}-\varepsilon\cdot c_2(x^d(\theta,\varepsilon),\theta) & \textit{if}\ \ \varepsilon\ge e^d.\end{cases}$$

Therefore, the ex-ante efficiency is equivalent to $\partial W^*(\theta,\varepsilon)/\partial\theta<0$ for any (θ,ε). Due to the properties that $\partial x^r(\theta,\varepsilon)/\partial\theta=-c_{12}(x^r(\theta,\varepsilon),\theta)/c_{11}(x^r(\theta,\varepsilon),\theta)>0$ and similarly $\partial x^d(\theta,\varepsilon)/\partial\theta>0$, one set of sufficient conditions for ex-ante efficiency is that the daily cost c_s is sufficiently large with $r<d\le c_s$ and $c_s\ge -e^d\cdot c_1(x^{B^*}(\theta),\theta)$ for any θ. As implied by (A.15), $c_1(x^{B^*}(\theta),\theta)\equiv\kappa_1$ is constant for any θ. Since the constant cutoff uncertainty is $e^d=-d/\kappa_1$, which is implied by (A.3) and (A.19), then $c_s\ge -e^d\cdot c_1(x^{B^*}(\theta),\theta)$ reduces to $c_s\ge d$. Therefore, one sufficient condition for ex-ante efficiency is $r<d\le c_s$.

Proof of Lemma 2

Let $\widetilde{\theta}=\delta\theta$, which is distributed as $\widetilde{F}_{\widetilde{\Theta}}(\cdot)$ on $[\underline{\widetilde{\theta}},\overline{\widetilde{\theta}}]=[\delta\underline{\theta},\delta\overline{\theta}]$. Denote by $(\widetilde{X}^B,\widetilde{P}^B,\widetilde{I}^R,\widetilde{I}^D,\widetilde{X}^r,\widetilde{X}^d)$ the endogenous variables generated by the structure $\widetilde{\mathcal{M}}$. We will show $(\widetilde{X}^B,\widetilde{P}^B,\widetilde{I}^R,\widetilde{I}^D,\widetilde{X}^r,\widetilde{X}^d,\widetilde{S})=(X^B,P^B,I^R,I^D,X^r,X^d,S)$, which implies the observational equivalency.

Recall that in equilibrium the bid days is strictly increasing in cost type. Let $\widetilde{X}^B(\cdot)=X^B(\cdot/\delta)$. As shown by (A.15), in equilibrium both $c_1(X^B(\theta),\theta)=\kappa_1$ and $\widetilde{c}_1(\widetilde{X}^B,\widetilde{\theta})=\widetilde{\kappa}_1$

are constant for any θ and any $\widetilde{\theta}$, respectively. Then,

$$\widetilde{\kappa}_1 = \widetilde{c}_1(\widetilde{X}^B, \widetilde{\theta}) = \widetilde{\theta}\widetilde{c}_{o,1}(\widetilde{X}^B(\widetilde{\theta})) = \theta c_{o,1}(\widetilde{X}^B(\widetilde{\theta})) = \theta c_{o,1}(X^B(\theta)) = c_1(X^B(\theta), \theta) = \kappa_1,$$

where the third equality follows $\widetilde{c}_o(\cdot) = c_o(\cdot)/\delta$. Denote by $\widetilde{e}^r$ and $\widetilde{e}^d$ the two cutoff levels of uncertainty for early and delay completion, respectively. From (A.3) we obtain $\widetilde{e}^r = e^r$ and $\widetilde{e}^d = e^d$, which implies

$$\widetilde{I}^R = I^R \text{ and } \widetilde{I}^D = I^D.$$

As a result,

$$r = -\varepsilon\widetilde{c}_1(\widetilde{X}^r(\widetilde{\theta}, \varepsilon), \widetilde{\theta}) = -\varepsilon\widetilde{\theta}\widetilde{c}_{o,1}(\widetilde{X}^r(\widetilde{\theta}, \varepsilon)) = -\varepsilon\theta c_{o,1}(\widetilde{X}^r(\widetilde{\theta}, \varepsilon)),$$

$$d = -\varepsilon\widetilde{c}_1(\widetilde{X}^d(\widetilde{\theta}, \varepsilon), \widetilde{\theta}) = -\varepsilon\widetilde{\theta}\widetilde{c}_{o,1}(\widetilde{X}^d(\widetilde{\theta}, \varepsilon)) = -\varepsilon\theta c_{o,1}(\widetilde{X}^d(\widetilde{\theta}, \varepsilon)).$$

Due to $c_{o,1}(\cdot) < 0$ implied by the assumption $0 > c_1(\cdot, \theta) = \theta c_{o,1}(\cdot)$, it follows

$$\widetilde{X}^r(\widetilde{\theta}, \varepsilon) = X^r(\theta, \varepsilon) \text{ and } \widetilde{X}^d(\widetilde{\theta}, \varepsilon) = X^d(\theta, \varepsilon).$$

Last, we need to show $\widetilde{P}(\widetilde{\theta}) = P(\theta)$. Note that $\widetilde{F}_{\widetilde{\Theta}}(\widetilde{\theta}) = F_\Theta(\theta)$ due to $\widetilde{F}_{\widetilde{\Theta}}(\cdot) = F_\Theta(\cdot/\delta)$. According to the equilibrium bid cost (A.28), we obtain $\widetilde{P}^B(\widetilde{\theta}) = P^B(\theta)$ since

$$\begin{aligned}
\widetilde{P}^B(\widetilde{\theta}) &= \mathbb{E}_\varepsilon\left[\varepsilon \cdot \widetilde{c}\left(\widetilde{X}^A(\widetilde{X}^B(\widetilde{\theta}), \widetilde{\theta}, \varepsilon\right) + K\left(\widetilde{X}^A(\widetilde{X}^B(\widetilde{\theta}), \widetilde{\theta}, \varepsilon), \widetilde{X}^B(\widetilde{\theta}), r, d\right)\right] \\
&+ \int_{\widetilde{\theta}}^{\widetilde{\overline{\theta}}} \mathbb{E}_\varepsilon\left[\varepsilon \cdot \widetilde{c}_o\left(\widetilde{X}^A(\widetilde{X}^B(\widetilde{t}), \widetilde{t}, \varepsilon), \widetilde{t}\right)\right]\left[\frac{1 - F_{\widetilde{\Theta}}(\widetilde{t})}{1 - F_{\widetilde{\Theta}}(\widetilde{\theta})}\right]^{N-1} d\widetilde{t} \\
&= \mathbb{E}_\varepsilon\left[\varepsilon \cdot c\left(X^A(X^B(\theta), \theta, \varepsilon\right) + K\left(X^A(X^B(\theta), \theta, \varepsilon), X^B(\theta), r, d\right)\right] \\
&+ \int_{\theta}^{\overline{\theta}} \mathbb{E}_\varepsilon\left[\varepsilon \cdot c_o\left(X^A(t, \varepsilon), t\right)/\delta\right]\left[\frac{1 - F_\Theta(t)}{1 - F_\Theta(\theta)}\right]^{N-1} d(\delta t) = P^B(\theta).
\end{aligned}$$

Therefore, $\widetilde{S} = \widetilde{P}^B(\widetilde{\theta}) + c_u\widetilde{X}^B(\widetilde{\theta}) = P^B(\theta) + c_uX^B(\theta) = S$. The proof is complete.

Proof of Proposition 3

Recall the recovered $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$ in (26) is given by

(A.31)

$$\beta_0 = \alpha_0 \frac{m_\varepsilon \beta_1^3}{2\beta_2 \alpha_1^3} + \frac{\beta_1^2}{\beta_2} - \frac{\beta_1^2(m_\varepsilon \beta_1^2 + 2\kappa_0 \alpha_1^2 \beta_2)}{4\alpha_1^2 \beta_2^2} - r\frac{\beta_1^2}{2\beta_2}\frac{\beta_1 + \beta_2\mu_r}{\beta_1 + \beta_2\mu_B} F(e^r) - d\frac{\beta_1^2}{2\beta_2}\frac{\beta_1 + \beta_2\mu_d}{\beta_1 + \beta_2\mu_B}(1 - F(e^d)),$$

$$\begin{aligned}
\beta_1 &= -4\kappa_1\alpha_1\alpha_2, \;\; \beta_3 = -\beta_1(c_u + \kappa_0) + 2\beta_1\left[rF(e^r) + d(1 - F(e^d))\right] - 2\alpha_1^2\beta_2\kappa_2/\beta_1, \\
\beta_2 &= -8\kappa_1\alpha_2^2, \;\; \beta_4 = -\beta_2\left\{c_u - \left[rF(e^r) + d(1 - F(e^d))\right]\right\} - \kappa_0\beta_2/2 - \alpha_1^2\beta_2^2\kappa_2/\beta_1^2.
\end{aligned}$$

In addition, (24) implies

$$X^B = \frac{\kappa_1}{2\alpha_2\theta} - \frac{\alpha_1}{2\alpha_2} \quad \text{for on-time completion,} \tag{A.32}$$

$$X^r = \frac{-r\varepsilon^{-1}\theta^{-1} - \alpha_1}{2\alpha_2} \quad \text{for early completion,} \tag{A.33}$$

$$X^d = \frac{-d\varepsilon^{-1}\theta^{-1} - \alpha_1}{2\alpha_2} \quad \text{for late completion.} \tag{A.34}$$

Then, we obtain

$$\alpha_2 = \frac{\alpha_1\beta_2}{2\beta_1}, \tag{A.35}$$

$$\kappa_1\alpha_1^2 = -\frac{\beta_1^2}{2\beta_2}. \tag{A.36}$$

Due to the one-to-one mapping between θ and X^B, (A.32) implies

$$\underline{X}^B = \frac{\kappa_1}{2\alpha_2\underline{\theta}} - \frac{\alpha_1}{2\alpha_2}. \tag{A.37}$$

where $\underline{X}^B$ is the lower bound of the bid days. Combining (A.35), (A.36), (A.37) with part (b) in Assumption 2 can identify (α_1, α_2)

$$\alpha_1 = -\beta_1 \left[2\underline{\theta}\beta_2 \left(\beta_1 + \beta_2 \underline{X}^B\right)\right]^{-1/3}, \tag{A.38}$$

$$\alpha_2 = \frac{\alpha_1 \beta_2}{2\beta_1}. \tag{A.39}$$

As a result, we can recover contractor's corresponding type and uncertainty

$$\begin{aligned} \theta &= -\frac{\beta_1^2}{2\beta_2\alpha_1^2\left(\alpha_1 + 2\alpha X^B\right)} && \text{for any contract,} \\ \varepsilon &= -\frac{r}{\theta(\alpha_1 + 2\alpha_2 X^r)} && \text{for early completion contract,} \\ \varepsilon &= -\frac{d}{\theta(\alpha_1 + 2\alpha_2 X^d)} && \text{for delay completion contract.} \end{aligned}$$

Using the identified types θ recovers the type distribution $F_\Theta(\cdot)$ on its support $\mathcal{S}_\Theta$.

For those contracts which are completed on time, the actual working days equals the bid days, which does not depend on uncertainty. Consequently, one cannot recover the uncertainty associated with on-time completion contracts. This implies that the uncertainty distribution $F(\cdot)$ cannot be identified on its entire support $\mathcal{S}_\varepsilon$. In other words, using X^r and X^d we can back out the corresponding uncertainty. Hence, we identify the truncated cumulative distribution function denoted by $G(\cdot)$ of ε on its partial support $\widetilde{\mathcal{S}}_\varepsilon = \mathcal{S}_r \cup \mathcal{S}_d$ with $\mathcal{S}_r = \{\varepsilon : \varepsilon \leq \varepsilon^r\}$ and $\mathcal{S}_d = \{\varepsilon : \varepsilon \geq \varepsilon^d\}$. As a result, the uncertainty distribution $F(\cdot)$ is identified on $\widetilde{\mathcal{S}}_\varepsilon$ as

$$F(\varepsilon) = G(\varepsilon)F(\varepsilon^r) \text{ if } \varepsilon \in \mathcal{S}_r, \text{ and } F(\varepsilon) = G(\varepsilon)(1 - F(\varepsilon^d)) \text{ if } \varepsilon \in \mathcal{S}_d.$$

Under the assumption that the mean of the uncertainty m_ε is known, we use (A.31) to identify α_0 as

$$\alpha_0 = \frac{2\beta_2\alpha_1^3}{m_\varepsilon \beta_1^3}\left(\beta_0 + \kappa_3 + \frac{\beta_1^2(m_\varepsilon \beta_1^2 + 2\kappa_0\alpha_1^2\beta_2)}{4\alpha_1^2\beta_2^2} - \frac{\beta_1^2}{\beta_2}\right), \tag{A.40}$$

where κ_3 is given by

$$\kappa_3 = r\frac{\beta_1^2(\beta_1+\beta_2\mu_r)}{2\beta_2(\beta_1+\beta_2\mu_B)}F(e^r) + d\frac{\beta_1^2(\beta_1+\beta_2\mu_d)}{2\beta_2(\beta_1+\beta_2\mu_B)}(1-F(e^d)),$$

which is obviously identified.

As explained in the text, the equilibrium bid cost (A.28) provides no additional identification power. This is because the bid cost can be written as a known function of all identified objects as below

$$\begin{aligned} P^B &= V - c_u X^B \\ &+ \int_{y\geq X^B}\left[\frac{2\beta_2}{\beta_1^3}\left\{\kappa_3+\frac{\beta_1^2\kappa_0}{2\beta_2}-\frac{\beta_1^2}{\beta_2}+\beta_0\right\}+\left\{\frac{2\beta_2\kappa_3}{\beta_1^5}(\beta_1+\beta_2 y)^2-\frac{\kappa_0}{\beta_1}-\frac{\kappa_0\beta_2\left[2\beta_1 y+\beta_2 y^2\right]}{\beta_1^3}\right\}\right] \\ &\left[\frac{1-F_{X^B}(y)}{1-F_{X^B}(X^B)}\right]^{N-1}\frac{\beta_1^3}{2(\beta_1+\beta_2 y)^2}dy. \end{aligned}$$

The proof is complete.

Proof of Corollary 2

We briefly show the efficiency of A+B contracts under two alternative time incentives $K_Q(\cdot)$ and $K_P(\cdot)$, respectively, since the proofs are very similar to that of Proposition 1 and 2.

Case 1: A+B contracts under $K_Q(\cdot)$ First, the optimal actual working days for any x^B under $K_Q(\cdot)$ is given by

$$\widetilde{x}_Q^{A^*}(x^B,\theta,\varepsilon) = \underset{x^A}{\operatorname{argmin}}\left\{\varepsilon\cdot c(x^A,\theta)+K_Q(x^A,x^B,r,d,\delta)\right\}. \tag{A.41}$$

Similar to the proof of Lemma 1, the second-stage optimal actual working days given $(x^B, \theta, \varepsilon)$ satisfies

$$\widetilde{x}_Q^{A^*}(x^B, \theta, \varepsilon) = \begin{cases} x^r(\theta, \varepsilon) & \textit{if} \;\; \varepsilon \leq \varepsilon^r(\theta, x^B), \\ x^B & \textit{if} \;\; \varepsilon \in [\varepsilon^r(\theta, x^B), \varepsilon^d(\theta, x^B)], \\ x_Q^d(\theta, \varepsilon, x^B) & \textit{if} \;\; \varepsilon \geq \varepsilon^d(\theta, x^B), \end{cases} \tag{A.42}$$

where $\varepsilon^r(\theta, x^B) < \varepsilon^d(\theta, x^B)$ and $x^r(\theta, \varepsilon) \leq x^B \leq x_Q^d(\theta, \varepsilon, x^B)$. Note that the two cutoff levels of uncertainty and the early working days under $K_Q(\cdot)$ are the same as under $K(\cdot)$. Due to the quadratic penalty for late completion, the late working days $x_Q^d(\theta, \varepsilon, x^B)$ under $K_Q(\cdot)$ is given by

$$-\varepsilon \cdot c_1(x_Q^d(\theta, \varepsilon, x^B), \theta) = d\left[1 + \delta \cdot (x_Q^d(\theta, \varepsilon, x^B) - x^B)\right]. \tag{A.43}$$

As explained for the ex-post inefficiency in Proposition 2, the actual working days cannot always equal the unique socially optimal working days due to early or late completion. As a result, the A+B contract under $K_Q(\cdot)$ cannot be ex-post efficient in the presence of ex-post uncertainty.

Second, we prove $dx_Q^{B^*}(\theta)/d\theta > 0$, where $x_Q^{B^*}(\theta)$ is the equilibrium bid days for type θ under $K_Q(\cdot)$. As in the proof of Proposition 1, $x_Q^{B^*}(\theta)$ is determined separately from the choice of score through

(A.44)

$$x_Q^{B^*}(\theta) = arg\min_{x^B} \left\{ c_u x^B + \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}_Q^{A^*}(x^B, \theta, \varepsilon), \theta) + K_Q(\widetilde{x}_Q^{A^*}(x^B, \theta, \varepsilon), x^B, r, d, \delta) \Big] \right\}.$$

Similarly, using the first-order condition with respect to x^B in (A.44) gives rise to

(A.45)
$$0 = c_u - \int_{\varepsilon \le \varepsilon^r(\theta, x^B)} r f(\varepsilon) d\varepsilon + \int_{\varepsilon^r(\theta, x^B)}^{\varepsilon^d(\theta, x^B)} \varepsilon \cdot c_1(x^B, \theta) f(\varepsilon) d\varepsilon + \int_{\varepsilon \ge \varepsilon^d(\theta, x^B)} \varepsilon \cdot c_1(x_Q^d(\theta, \varepsilon, x^B), \theta) f(\varepsilon) d\varepsilon.$$

Combining (A.43) with (A.45) implies that $x_Q^{B^*} = x_Q^{B^*}(\theta)$ satisfies

$$\begin{aligned} -c_u =& c_1(x_Q^{B^*}, \theta) \int_{\varepsilon^r(\theta, x_Q^{B^*})}^{\varepsilon^d(\theta, x_Q^{B^*})} \varepsilon f(\varepsilon) d\varepsilon - rF(\varepsilon^r(\theta, x_Q^{B^*})) - d\left[1 - F(\varepsilon^d(\theta, x_Q^{B^*}))\right] \\ & + d \cdot \delta \cdot x_Q^{B^*}\left[1 - F(\varepsilon^d(\theta, x_Q^{B^*}))\right] - d \cdot \delta \cdot \int_{\varepsilon \ge \varepsilon^d(\theta, x_Q^{B^*})} x_Q^d(\theta, \varepsilon, x_Q^{B^*}) f(\varepsilon) d\varepsilon. \end{aligned} \quad \text{(A.46)}$$

As expected, when δ approaches to zero, $x_Q^{B^*}(\theta)$ is closer to $x^{B^*}(\theta)$ by comparing (A.46) with (A.13).

Denote $x_Q^d = x_Q^d(\theta, \varepsilon) = x_Q^d\left(\theta, \varepsilon, x_Q^{B^*}(\theta)\right)$ for simplicity. Taking the first derivative of both sides of (A.45) with respect to θ implies

(A.47)
$$0 = \int_{\varepsilon^r(\theta, x_Q^{B^*})}^{\varepsilon^d(\theta, x_Q^{B^*})} \varepsilon \left[c_{11}(x_Q^{B^*}, \theta)\frac{dx_Q^{B^*}}{d\theta} + c_{12}(x_Q^{B^*}, \theta)\right] f(\varepsilon) d\varepsilon + \int_{\varepsilon \ge \varepsilon^d(\theta, x_Q^{B^*})} \varepsilon \left[\widetilde{c}_{11}\frac{\partial x_Q^d}{\partial \theta} + \widetilde{c}_{12}\right] f(\varepsilon) d\varepsilon,$$

where $\widetilde{c}_{11} = c_{11}\left(x_Q^d\left(\theta, \varepsilon, x_Q^{B^*}\right), \theta\right)$ and $\widetilde{c}_{12} = c_{12}\left(x_Q^d\left(\theta, \varepsilon, x_Q^{B^*}\right), \theta\right)$. In addition, taking the first derivative of both sides of (A.43) at $x^B = x^{B^*}(\theta)$ with respect to θ leads to

$$-\varepsilon \cdot \left[c_{11}(x_Q^d(\theta, \varepsilon), \theta)\frac{\partial x_Q^d(\theta, \varepsilon)}{\partial \theta} + c_{12}(x_Q^d(\theta, \varepsilon), \theta)\right] = d \cdot \delta \cdot \left[\frac{\partial x_Q^d(\theta, \varepsilon)}{\partial \theta} - \frac{dx_Q^{B^*}(\theta)}{d\theta}\right],$$

which implies

$$\frac{\partial x_Q^d(\theta, \varepsilon)}{\partial \theta} = \frac{-\varepsilon \cdot c_{12}(x_Q^d(\theta, \varepsilon), \theta)}{\delta \cdot d + \varepsilon \cdot c_{11}(x_Q^d(\theta, \varepsilon), \theta)} + \frac{\delta \cdot d \cdot \frac{dx_Q^{B^*}(\theta)}{d\theta}}{\delta \cdot d + \varepsilon \cdot c_{11}(x_Q^d(\theta, \varepsilon), \theta)}. \quad \text{(A.48)}$$

Suppose $dx_Q^{B^*}(\theta)/d\theta \leq 0$, then,

$$\frac{\partial x_Q^d(\theta,\varepsilon)}{\partial\theta} \leq \frac{-\varepsilon \cdot c_{12}(x_Q^d(\theta,\varepsilon),\theta)}{\delta \cdot d + \varepsilon \cdot c_{11}(x_Q^d(\theta,\varepsilon),\theta)} < \frac{-\varepsilon \cdot c_{12}(x_Q^d(\theta,\varepsilon),\theta)}{\varepsilon \cdot c_{11}(x_Q^d(\theta,\varepsilon),\theta)},$$

which implies $\varepsilon \cdot \left[c_{11}(x_Q^d(\theta,\varepsilon),\theta)\frac{\partial x_Q^d(\theta,\varepsilon)}{\partial\theta} + c_{12}(x_Q^d(\theta,\varepsilon),\theta)\right] < 0$. Combining with (A.47) implies

$$c_{11}(x_Q^{B^*}(\theta),\theta)\frac{dx_Q^{B^*}(\theta)}{d\theta} + c_{12}(x_Q^{B^*}(\theta),\theta) > 0. \tag{A.49}$$

Due to $c_{11}(\cdot,\cdot) > 0$ and $c_{12}(\cdot,\cdot) < 0$, we obtain

$$\frac{dx_Q^{B^*}(\theta)}{d\theta} > -\frac{c_{12}(x_Q^{B^*}(\theta),\theta)}{c_{11}(x_Q^{B^*}(\theta),\theta)} > 0,$$

which is a contradiction. Hence,

$$dx_Q^{B^*}(\theta)/d\theta > 0. \tag{A.50}$$

Accordingly, by combining with (A.48) we obtain

$$\partial x_Q^d(\theta,\varepsilon)/\partial\theta > 0. \tag{A.51}$$

Now, we are in a position to use (A.50) and (A.51) to prove that the A+B contract under $K_Q(\cdot)$ can be ex-ante efficiency. However, it seems less likely to be ex-ante efficient than under $K(\cdot)$. Intuitively, a more flexible time incentive includes more parameters, requiring more constraints for the ex-ante efficiency. Note that the cutoff levels of uncertainty $\varepsilon_Q^r(\theta) = \varepsilon^r(\theta, x_Q^{B^*}(\theta))$ and $\varepsilon_Q^d(\theta) = \varepsilon^d(\theta, x_Q^{B^*}(\theta))$ depend on θ. This is because the bid days associated with $K_Q(\cdot)$ does not have a relationship similar to (A.15) associated with $K(\cdot)$, while this relationship is key to the constancy of cutoff levels of uncertainty under $K(\cdot)$. Consequently, it may entail more restrictive constraints for the ex-ante efficiency of A+B contracts under $K_Q(\cdot)$.

The social welfare in equilibrium under $K_Q(\cdot)$ is given by

$$W_Q^*(\theta,\varepsilon) = \begin{cases} V_c - \varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) - c_s x^r(\theta,\varepsilon) & \textit{if } \varepsilon \le \varepsilon_Q^r(\theta), \\ V_c - \varepsilon \cdot c(x_Q^{B^*}(\theta),\theta) - c_s x_Q^{B^*}(\theta) & \textit{if } \varepsilon_Q^r \le \varepsilon \le \varepsilon_Q^d(\theta), \\ V_c - \varepsilon \cdot c(x_Q^d(\theta,\varepsilon),\theta) - c_s x_Q^d(\theta,\varepsilon) & \textit{if } \varepsilon \ge \varepsilon_Q^d(\theta). \end{cases} \tag{A.52}$$

The ex-ante efficiency requires $W_Q^*(\theta,\varepsilon) > W_Q^*(\theta',\varepsilon)$ for any $\theta < \theta'$. Without loss of generality, we assume $\varepsilon_Q^r(\theta) < \varepsilon_Q^r(\theta')$, $\varepsilon_Q^d(\theta) < \varepsilon_Q^d(\theta')$, and $\varepsilon_Q^r(\theta') < \varepsilon_Q^d(\theta)$.[25] To analyze the ex-ante efficiency under $K_Q(\cdot)$ we need to consider five cases: (i) $\varepsilon \le \varepsilon_Q^r(\theta)$, (ii) $\varepsilon_Q^r(\theta) < \varepsilon \le \varepsilon_Q^r(\theta')$, (iii) $\varepsilon_Q^r(\theta') < \varepsilon \le \varepsilon_Q^d(\theta)$, (iv) $\varepsilon_Q^d(\theta) < \varepsilon \le \varepsilon_Q^d(\theta')$, and (iv) $\varepsilon \ge \varepsilon_Q^d(\theta')$.

On the one hand, for any ε in case (i), (iii), or (v), contractors with either θ or θ' will response in the same manner in terms of whether the project is finished early, on time, or late. Taking first derivative of $W_Q^*(\theta,\varepsilon)$ with respect to θ and using (A.43) lead to

(A.53)

$$\frac{\partial W_Q^*(\theta,\varepsilon)}{\partial\theta} = \begin{cases} (r - c_s)\frac{\partial x^r(\theta,\varepsilon)}{\partial\theta} - \varepsilon \cdot c_2(x^r(\theta,\varepsilon),\theta) & \textit{under (i)}, \\ -\left(\varepsilon \cdot c_1(x_Q^{B^*}(\theta),\theta) + c_s\right)\frac{dx_Q^{B^*}(\theta)}{d\theta} - \varepsilon \cdot c_2(x_Q^{B^*}(\theta),\theta) & \textit{under (iii)}, \\ \left[d\delta\left(x_Q^d(\theta,\varepsilon) - x_Q^{B^*}(\theta)\right) + d - c_s\right]\frac{\partial x_Q^d(\theta,\varepsilon)}{\partial\theta} - \varepsilon \cdot c_2(x_Q^d(\theta,\varepsilon),\theta) & \textit{under (v)}. \end{cases}$$

On the other hand, under case (ii) a contractor will complete the project on time with type θ while complete the project early with type θ'. Under case (iv) a contractor will complete the project on time with type θ' while delay the completion with type θ. Let

[25] If (A.49) holds, under the assumption $c_1(\cdot,\cdot) < 0$, these relationships between cutoff levels are satisfied using the results similar to (A.17)-(A.18) and the result $\varepsilon_Q^r(\theta') = -r \cdot [c_1(x_Q^{B^*}(\theta'),\theta')]^{-1} < -d \cdot [c_1(x_Q^{B^*}(\theta'),\theta')]^{-1} < -d \cdot [c_1(x_Q^{B^*}(\theta),\theta)]^{-1} = \varepsilon_Q^d(\theta)$.

$\Delta W_Q(\theta,\theta',\varepsilon) = W_Q^*(\theta,\varepsilon) - W_Q^*(\theta',\varepsilon)$. Then,

(A.54)

$$\Delta W_Q(\theta,\theta',\varepsilon) = \begin{cases} -\varepsilon \cdot c(x_Q^{B^*}(\theta),\theta) - c_s x_Q^{B^*}(\theta) + \varepsilon \cdot c(x^r(\theta'),\theta') + c_s x^r(\theta') & \textit{under (ii)}, \\ -\varepsilon \cdot c(x_Q^d(\theta,\varepsilon),\theta) - c_s x_Q^d(\theta,\varepsilon) + \varepsilon \cdot c(x_Q^{B^*}(\theta'),\theta') + c_s x_Q^{B^*}(\theta') & \textit{under (iv)}. \end{cases}$$

Hence, the ex-ante efficiency holds if, for any $\theta < \theta'$ and ε, $\partial W_Q^*(\theta,\varepsilon)/\partial\theta < 0$ in (A.53) and $\Delta W_Q(\theta,\theta',\varepsilon) > 0$ in (A.54). As shown in Proposition 2, when $r < d \leq c_s$, the A+B contract must be ex-ante efficient under $K(\cdot)$. However, it is not necessarily ex-ante efficient under $K_Q(\cdot)$. For example, when $r < d \leq c_s$, $\partial W_Q^*(\theta,\varepsilon)/\partial\theta < 0$ may not hold under (iii) and (v) according to (A.50) and (A.51), respectively, not to mention $\Delta W_Q(\theta,\theta',\varepsilon) > 0$ in (A.54).

Case 2: A+B contracts under $K_P(\cdot)$ First, the optimal actual working days for any x^B under $K_P(\cdot)$ is given by

$$\widetilde{x}_P^{A^*}(x^B,\theta,\varepsilon) = \operatorname*{argmin}_{x^A} \left\{\varepsilon \cdot c(x^A,\theta) + K_P(x^A,x^B,r,d,\widetilde{d},\rho)\right\}. \tag{A.55}$$

Similar to the proof of Lemma 1, the second-stage optimal actual working days given (x^B,θ,ε) satisfies

$$\widetilde{x}_P^{A^*}(x^B,\theta,\varepsilon) = \begin{cases} x^r(\theta,\varepsilon) & \textit{if } \varepsilon \leq \varepsilon^r(\theta,x^B), \\ x^B & \textit{if } \varepsilon \in [\varepsilon^r(\theta,x^B),\varepsilon^d(\theta,x^B)], \\ x^d(\theta,\varepsilon) & \textit{if } \varepsilon \in [\varepsilon^d(\theta,x^B),\varepsilon^d(\theta,\rho x^B)], \\ \rho x^B & \textit{if } \varepsilon \in [\varepsilon^d(\theta,\rho x^B),\varepsilon^{\widetilde{d}}(\theta,\rho x^B)], \\ x^{\widetilde{d}}(\theta,\varepsilon) & \textit{if } \varepsilon \geq \varepsilon^{\widetilde{d}}(\theta,\rho x^B), \end{cases} \tag{A.56}$$

where

$$\varepsilon^{d}(\theta, \rho x^{B}) = \frac{-d}{c_1(\rho x^{B}, \theta)}, \quad \varepsilon^{\widetilde{d}}(\theta, \rho x^{B}) = \frac{-\widetilde{d}}{c_1(\rho x^{B}, \theta)} \tag{A.57}$$

with

$$\varepsilon^{r}(\theta, x^{B}) < \varepsilon^{d}(\theta, x^{B}) < \varepsilon^{d}(\theta, \rho x^{B}) < \varepsilon^{\widetilde{d}}(\theta, \rho x^{B}), \tag{A.58}$$

and $x^{\widetilde{d}}(\theta, \varepsilon)$ satisfies

$$-\varepsilon \cdot c_1(x^{\widetilde{d}}(\theta, \varepsilon), \theta) = \widetilde{d}, \tag{A.59}$$

with $x^{r}(\theta, \varepsilon) \leq x^{B} \leq x^{d}(\theta, \varepsilon) \leq \rho x^{B} \leq x^{\widetilde{d}}(\theta, \varepsilon)$.

As explained for the ex-post inefficiency in Proposition 2, the actual working days cannot always equal the unique socially optimal working days due to early or late completion. As a result, the A+B contract under $K_P(\cdot)$ cannot be ex-post efficient in the presence of ex-post uncertainty.

Second, we prove

$$dx_P^{B^*}(\theta)/d\theta > 0, \tag{A.60}$$

where $x_P^{B^*}(\theta)$ is the equilibrium bid days for type θ under $K_P(\cdot)$. As in the proof of Proposition 1, the bid days $x_P^{B^*}(\theta)$ is determined separately from the choice of score through

(A.61)

$$x_P^{B^*}(\theta) = \arg\min_{x^B} \left\{ c_u x^B + \mathbb{E}_\varepsilon \Big[\varepsilon \cdot c(\widetilde{x}_P^{A^*}(x^B, \theta, \varepsilon), \theta) + K_P(\widetilde{x}_P^{A^*}(x^B, \theta, \varepsilon), x^B, r, d, \widetilde{d}, \rho) \Big] \right\}.$$

Similarly, using the first-order condition with respect to x^B in (A.61) gives rise to

$$0 = c_u + c_1(x^B,\theta)\int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon f(\varepsilon)d\varepsilon - rF(\varepsilon^r(\theta,x^B)) - d\big[F(\varepsilon^d(\theta,\rho x^B)) - F(\varepsilon^d(\theta,x^B))\big]$$

(A.62)

$$+\rho\cdot c_1(\rho x^B,\theta)\int_{\varepsilon^d(\theta,\rho x^B)}^{\varepsilon^{\widetilde{d}}(\theta,\rho x^B)} \varepsilon f(\varepsilon)d\varepsilon + d(\rho-1)[1-F(\varepsilon^d(\theta,\rho x^B)] - \widetilde{d}\rho[1-F(\varepsilon^{\widetilde{d}}(\theta,\rho x^B)]$$

$$= c_u + c_1(x^B,\theta)\int_{\varepsilon^r(\theta,x^B)}^{\varepsilon^d(\theta,x^B)} \varepsilon f(\varepsilon)d\varepsilon - rF(\varepsilon^r(\theta,x^B)) - d\big[1 - F(\varepsilon^d(\theta,x^B))\big]$$

(A.63)

$$+\rho\cdot(d-\widetilde{d})[1-F(\widetilde{\varepsilon})],$$

where $\widetilde{\varepsilon}\in[\varepsilon^d(\theta,\rho x^B),\varepsilon^{\widetilde{d}}(\theta,\rho x^B)]$ is from the mean value theorem. As expected, when $\rho=1$ and $d=\widetilde{d}=d$, (A.63) reduce to (A.12) under $K(\cdot)$.

In addition, (A.62) implies that the equilibrium bid days $x_P^{B^*}=x_P^{B^*}(\theta)$ satisfies

$$-c_u = c_1(x_P^{B^*},\theta)\int_{\varepsilon^r(\theta,x_P^{B^*})}^{\varepsilon^d(\theta,x_P^{B^*})} \varepsilon f(\varepsilon)d\varepsilon - rF(\varepsilon^r(\theta,x_P^{B^*})) - d\big[F(\varepsilon^d(\theta,\rho x_P^{B^*})) - F(\varepsilon^d(\theta,x_P^{B^*}))\big]$$

(A.64)

$$+\rho\cdot c_1(\rho x_P^{B^*},\theta)\int_{\varepsilon^d(\theta,\rho x_P^{B^*})}^{\varepsilon^{\widetilde{d}}(\theta,\rho x_P^{B^*})} \varepsilon f(\varepsilon)d\varepsilon + d(\rho-1)[1-F(\varepsilon^d(\theta,\rho x_P^{B^*})] - \widetilde{d}\rho[1-F(\varepsilon^{\widetilde{d}}(\theta,\rho x_P^{B^*})].$$

Taking the first derivative of both sides of (A.64) with respect to θ implies

$$\begin{aligned} 0 \;=\;& \left[c_{11}(x_P^{B^*},\theta)\frac{\partial x_P^{B^*}}{\partial\theta} + c_{12}(x_P^{B^*},\theta)\right]\int_{\varepsilon^r(\theta,x_P^{B^*})}^{\varepsilon^d(\theta,x_P^{B^*})}\varepsilon f(\varepsilon)d\varepsilon \\ &+ \left[\rho^2 c_{11}(\rho x_P^{B^*},\theta)\frac{\partial x_P^{B^*}}{\partial\theta} + \rho c_{12}(\rho x_P^{B^*},\theta)\right]\int_{\varepsilon^d(\theta,\rho x_P^{B^*})}^{\varepsilon^{\widetilde{d}}(\theta,\rho x_P^{B^*})}\varepsilon f(\varepsilon)d\varepsilon. \end{aligned} \tag{A.65}$$

Due to $\varepsilon>0$, $c_{11}(\cdot,\cdot)>0$ and $c_{12}(\cdot,\cdot)<0$, it must be $dx_P^{B^*}(\theta)/d\theta>0$; otherwise, the last equality in (A.65) would be strictly negative.

Now, we are in a position to use (A.60) to prove that the A+B contract under $K_P(\cdot)$ can be ex-ante efficiency. However, it seems less likely to be ex-ante efficient than under $K(\cdot)$. As explained intuitively for the case with $K_Q(\cdot)$, a more flexible time incentive includes more parameters, requiring more constraints for the ex-ante efficiency. The social welfare in equilibrium under $K_P(\cdot)$ is given by

$$W_P^*(\theta,\varepsilon) = \begin{cases} V_c - \varepsilon \cdot c(x^r(\theta,\varepsilon),\theta) - c_s x^r(\theta,\varepsilon) & \textit{under (i)}, \\ V_c - \varepsilon \cdot c(x_P^{B^*}(\theta),\theta) - c_s x_P^{B^*}(\theta) & \textit{under (ii)}, \\ V_c - \varepsilon \cdot c(x^d(\theta,\varepsilon),\theta) - c_s x^d(\theta,\varepsilon) & \textit{under (iii)}, \\ V_c - \varepsilon \cdot c(\rho x_P^{B^*}(\theta),\theta) - c_s \rho x_P^{B^*}(\theta) & \textit{under (iv)}, \\ V_c - \varepsilon \cdot c(x^{\widetilde{d}}(\theta,\varepsilon),\theta) - c_s x^{\widetilde{d}}(\theta,\varepsilon) & \textit{under (v)}, \end{cases} \tag{A.66}$$

where (i) $\varepsilon \leq \varepsilon^r(\theta, x_P^{B^*}(\theta))$, (ii) $\varepsilon \in [\varepsilon^r(\theta, x_P^{B^*}(\theta)), \varepsilon^d(\theta, x_P^{B^*}(\theta))]$, (iii) $\varepsilon \in [\varepsilon^d(\theta, x_P^{B^*}(\theta)), \varepsilon^d(\theta, \rho x_P^{B^*}(\theta))]$, (iv) $\varepsilon \in [\varepsilon^d(\theta, \rho x_P^{B^*}(\theta)), \varepsilon^{\widetilde{d}}(\theta, \rho x_P^{B^*}(\theta))]$, and (v) $\varepsilon \geq \varepsilon^{\widetilde{d}}(\theta, \rho x_P^{B^*}(\theta))$.

The ex-ante efficiency requires $W_Q^*(\theta,\varepsilon) > W_Q^*(\theta',\varepsilon)$ for any $\theta < \theta'$. Similar to the case with $K_Q(\cdot)$, when contractors with either θ or θ' response in the same manner in terms of whether the project is finished early, on time, or late, the ex-ante efficiency implies $\partial W_P^*(\theta,\varepsilon)/\partial\theta < 0$, where

$$\frac{\partial W_P^*(\theta,\varepsilon)}{\partial\theta} = \begin{cases} (r - c_s)\frac{\partial x^r(\theta,\varepsilon)}{\partial\theta} - \varepsilon \cdot c_2(x^r(\theta,\varepsilon),\theta) & \textit{under (i)}, \\ -\left(\varepsilon \cdot c_1(x_P^{B^*}(\theta),\theta) + c_s\right)\frac{\partial x_P^{B^*}(\theta)}{\partial\theta} - \varepsilon \cdot c_2(x_P^{B^*}(\theta)\theta), & \textit{under (ii)}, \\ (d - c_s)\frac{\partial x^d(\theta,\varepsilon)}{\partial\theta} - \varepsilon \cdot c_2(x^d(\theta,\varepsilon),\theta) & \textit{under (iii)}, \\ -\rho\left(\varepsilon \cdot c_1(\rho x_P^{B^*}(\theta),\theta) + c_s\right)\frac{\partial x_P^{B^*}(\theta)}{\partial\theta} - \varepsilon \cdot c_2(\rho x_P^{B^*}(\theta),\theta) & \textit{under (iv)}, \\ (\widetilde{d} - c_s)\frac{\partial x^{\widetilde{d}}(\theta,\varepsilon)}{\partial\theta} - \varepsilon \cdot c_2(x^{\widetilde{d}}(\theta,\varepsilon),\theta) & \textit{under (v)}. \end{cases} \tag{A.67}$$

As shown in Proposition 2, when $r < d \leq c_s$, the A+B contract must be ex-ante efficient under $K(\cdot)$. However, it is not necessarily ex-ante efficient under $K_P(\cdot)$. For example, when $r < d \leq c_s$, $\partial W_P^*(\theta, \varepsilon)/\partial\theta < 0$ may not hold under (ii) and (iv) according to (A.60), not to mention $\partial W_P^*(\theta, \varepsilon)/\partial\theta < 0$ under (i), (iii), (iv) in (A.67). Moreover, there are more restrictive conditions for the ex-ante efficiency under the case in which, as in the previous scenario under $K_Q(\cdot)$, a contractor with θ complete the project in different completion manners from a contractor with θ'. The proof is complete.

Proof of Corollary 3

Since the identification strategy is similar to that in Proposition 3 and Corollary 1, we just provide a brief proof. Recall in (22)

$$c_1(X^B, \theta) = \frac{rF(e^r) + d - dF(e^d) - c_u}{\int_{e^r}^{e^d} \varepsilon dF(\varepsilon)} = \frac{\kappa_0}{\int_{e^r}^{e^d} \varepsilon dF(\varepsilon)} = \kappa_1.$$

Under Assumption 3, we obtain the bid days, early working days, and late working days, respectively, by

$$\text{(A.68)} \qquad X^B = \left(\frac{\kappa_1}{\widetilde{\alpha}\theta}\right)^{\frac{1}{\widetilde{\alpha}-1}}, \quad X^r = \left(\frac{-r}{\widetilde{\alpha}\theta\varepsilon}\right)^{\frac{1}{\widetilde{\alpha}-1}}, \quad \text{and} \quad X^d = \left(\frac{-d}{\widetilde{\alpha}\theta\varepsilon}\right)^{\frac{1}{\widetilde{\alpha}-1}}.$$

As a result,

$$X^r = X^B \varepsilon^{\frac{1}{1-\widetilde{\alpha}}} \left(\frac{-r}{\kappa_1}\right)^{\frac{1}{\widetilde{\alpha}-1}} \quad \text{and} \quad X^d = X^B \varepsilon^{\frac{1}{1-\widetilde{\alpha}}} \left(\frac{-d}{\kappa_1}\right)^{\frac{1}{\widetilde{\alpha}-1}}.$$

The independence between θ and ε implies

$$\text{(A.69)} \qquad \mathbb{E}(X^r) = \mathbb{E}(X^B)\mathbb{E}\left(\varepsilon^{\frac{1}{1-\widetilde{\alpha}}} | \varepsilon \leq e^r\right)(r)^{\frac{1}{\widetilde{\alpha}-1}}(-\kappa_1)^{\frac{1}{1-\widetilde{\alpha}}},$$

$$\text{(A.70)} \qquad \mathbb{E}(X^d) = \mathbb{E}(X^B)\mathbb{E}\left(\varepsilon^{\frac{1}{1-\widetilde{\alpha}}} | \varepsilon \geq e^d\right)(d)^{\frac{1}{\widetilde{\alpha}-1}}(-\kappa_1)^{\frac{1}{1-\widetilde{\alpha}}}.$$

Note that (A.26) can be rewritten as

$$V = c_u \cdot X^B + \mathbb{E}_\varepsilon\Big[\varepsilon \cdot c\left(X^A(X^B,\theta,\varepsilon),\theta\right) + K\left(X^A(X^B,\theta,\varepsilon),X^B,r,d\right)\Big].$$

Then, it can be shown that

$$\begin{aligned} V &= X^B\left\{c_u + \frac{\kappa_0}{\widetilde{\alpha}} - rF(e^r) - d(1-F(e^d)) + \frac{\widetilde{\alpha}-1}{\widetilde{\alpha}}\left[rF(e^r)\frac{\mathbb{E}(X^r)}{\mathbb{E}(X^B)} + d(1-F(e^d))\frac{\mathbb{E}(X^d)}{\mathbb{E}(X^B)}\right]\right\} \\ &= X^B\widetilde{\beta}, \end{aligned}$$

where the definition of $\widetilde{\beta}$ in the last equality should be apparent. Due to $v'(\theta) > 0$ and $dX^B(\theta)/d\theta > 0$, for any $\tau \in [0,1]$ we have

$$Q_V(\tau) = Q_{X^B}(\tau)\widetilde{\beta}.$$

Therefore, $\widetilde{\beta}$ can be identified by choosing any $\tau \in [0,1]$. We recover $\widetilde{\alpha}$ as

$$\widetilde{\alpha} = \frac{\kappa_0 - rF(e^r)\frac{\mathbb{E}(X^r)}{\mathbb{E}(X^B)} - d(1-F(e^d))\frac{\mathbb{E}(X^d)}{\mathbb{E}(X^B)}}{\widetilde{\beta} - c_u + rF(e^r) + d(1-F(e^d)) - rF(e^r)\frac{\mathbb{E}(X^r)}{\mathbb{E}(X^B)} - d(1-F(e^d))\frac{\mathbb{E}(X^d)}{\mathbb{E}(X^B)}}.$$

Next, since κ_1 is recovered as $\kappa_1 = \widetilde{\alpha}\underline{\theta}(\underline{X}^B)^{\widetilde{\alpha}-1}$ from (A.68), where $\underline{X}^B$ is the lower bound of the bid days, we can back out contractor's corresponding type and uncertainty

$$\begin{aligned} \theta &= \kappa_1\widetilde{\alpha}^{-1}\left(X^B\right)^{1-\widetilde{\alpha}} && \text{for any contract,} \\ \varepsilon &= -r\kappa_1^{-1}\left(X^r/X^B\right)^{1-\widetilde{\alpha}} && \text{for early completion contract,} \\ \varepsilon &= -d\kappa_1^{-1}\left(X^d/X^B\right)^{1-\widetilde{\alpha}} && \text{for delay completion contract.} \end{aligned}$$

Using the identified types θ recovers the type distribution $F_\Theta(\cdot)$ on its support $\mathcal{S}_\Theta$. Similar to Proposition 3, the uncertainty distribution $F(\cdot)$ is identified on $\widetilde{\mathcal{S}}_\varepsilon$. Using the same arguments as in Corollary 1, if the uncertainty distribution is parameterized, using the above recovered uncertainty can identify the set of parameters of the uncertainty distribution. The proof is complete.

B. Further Details for Empirical Application

We derive the expressions of $\{\beta_k(z_j)\}_{k=0}^4$ in (34). Using the prior estimates, we have

$$\widehat{e}^d(z_j) = exp(z_j\widehat{\psi}_d). \tag{B.1}$$

Combining with (A.3) implies

$$\frac{e^r(z_j)}{e^d(z_j)} = \frac{-r_j/c_1(x_{ji}^B, \theta_i)}{-d_j/c_1(x_{ji}^B, \theta_i)} = \frac{r_j}{d_j}.$$

Then,

$$\widehat{e}^r(z_j) = \frac{r_j}{d_j}\widehat{e}^d(z_j) = \frac{r_j}{d_j}exp(z_j\widehat{\psi}_d). \tag{B.2}$$

As a result,

$$\widehat{\kappa}_0(z_j) = r_j\widehat{F}(\widehat{e}^r(z_j)|z_j) + d_j\left[1 - \widehat{F}(\widehat{e}^d(z_j)|z_j)\right] - c_{u_j}, \tag{B.3}$$

$$\widehat{\kappa}_1(z_j) = \frac{\widehat{\kappa}_0(z_j)}{\int_{\widehat{e}^r(z_j)}^{\widehat{e}^d(z_j)} \varepsilon d\widehat{F}(\varepsilon|z_j)}. \tag{B.4}$$

Let

$$
\begin{aligned}
\widehat{\kappa}_4(z_j) &= r_j \frac{\alpha_1 + 2\alpha_2 \widehat{\mu}_r(z_j)}{\alpha_1 + 2\alpha_2 \widehat{\mu}_B(z_j)} \widehat{F}(\widehat{e}^r(z_j)|z_j) + d_j \frac{\alpha_1 + 2\alpha_2 \widehat{\mu}_d(z_j)}{\alpha_1 + 2\alpha_2 \widehat{\mu}_B(z_j)} (1 - \widehat{F}(\widehat{e}^d(z_j)|z_j)), \\
\widehat{\kappa}_5(z_j) &= \widehat{m}_\varepsilon(z_j) - \widehat{\kappa}_0(z_j)/\widehat{\kappa}_1(z_j), \\
\widehat{m}_\varepsilon(z_j) &= exp(\widehat{\mu} + \widehat{\sigma}(z_j)^2/2).
\end{aligned}
$$

Rearranging β in (26) implies

$$
\begin{aligned}
\beta_0(z_j) &= \alpha_1^2 \widehat{\kappa}_1(z_j) \left[\widehat{\kappa}_4(z_j) - 2 - \widehat{\kappa}_1(z_j)\widehat{\kappa}_5(z_j)\right] + 4\alpha_0\alpha_2\widehat{\kappa}_1(z_j) \left[\widehat{\kappa}_1(z_j)\widehat{\kappa}_5(z_j) + \widehat{\kappa}_0(z_j)\right], \\
\beta_1(z_j) &= -4\widehat{\kappa}_1(z_j)\alpha_1\alpha_2, \\
\beta_2(z_j) &= -8\widehat{\kappa}_1(z_j)\alpha_2^2, \\
\beta_3(z_j) &= 4\alpha_1\alpha_2\widehat{\kappa}_1(z_j) \left\{c_{u_j} + \widehat{\kappa}_0(z_j) + \widehat{\kappa}_4(z_j) - 2\left[r_j \widehat{F}(\widehat{e}^r(z_j)|z_j) + d_j(1 - \widehat{F}(\widehat{e}^d(z_j)|z_j))\right]\right\}, \\
\beta_4(z_j) &= 4\alpha_2^2\widehat{\kappa}_1(z_j) \left\{\widehat{\kappa}_0(z_j) + \widehat{\kappa}_4(z_j) + 2\left[c_{u_j} - r_j \widehat{F}(\widehat{e}^r(z_j)|z_j) - d_j(1 - \widehat{F}(\widehat{e}^d(z_j)|z_j))\right]\right\}.
\end{aligned}
$$

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